Route Planning in Road Networks

– simple, flexible, efficient –

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`http://algo2.iti.uka.de/schultes/hwy/`

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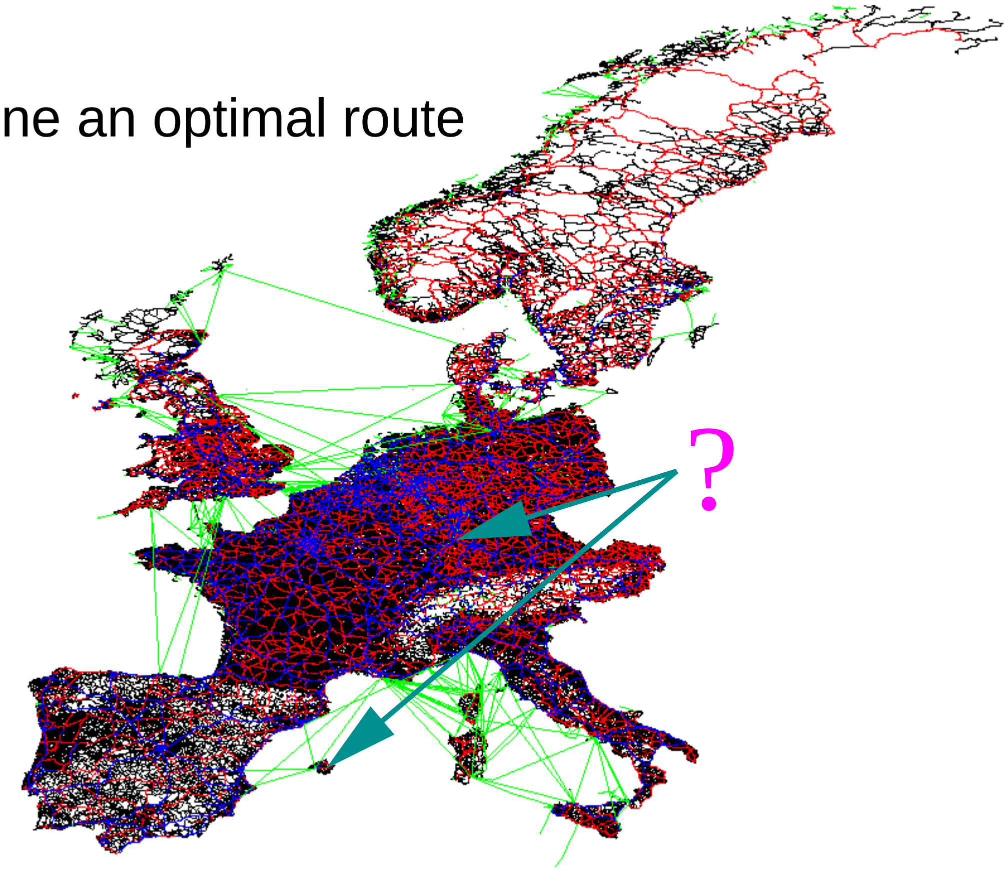
Route Planning

Task:

In a given road network, determine an optimal route

from a given source

to a given target



Applications:

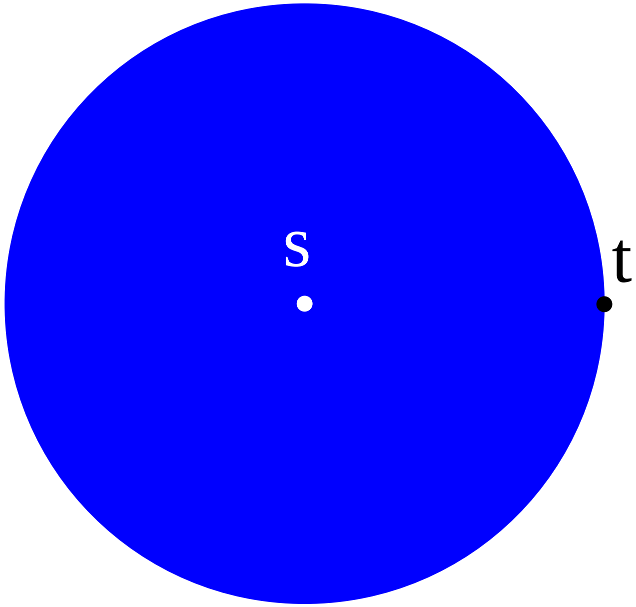
- route planning systems in the internet, car navigation systems,
- traffic simulation, logistics optimisation

DIJKSTRA's Algorithm

the classic solution [1959]

$O(n \log n + m)$ (with Fibonacci heaps)

Dijkstra



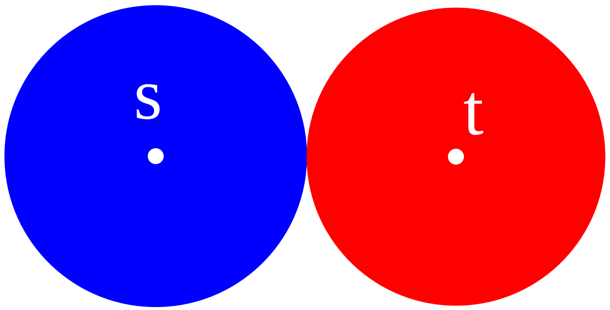
not practicable

for large graphs

(e.g. European road network:

$\approx$ 18 000 000 nodes)

bidirectional Dijkstra



improves the running time,

but still too slow

Speedup Techniques

that are faster than Dijkstra's algorithm

- require additional data (e.g., node coordinates)

  not always available!

AND / OR

- preprocess the graph and generate auxiliary data (e.g., 'signposts')

  can take a lot of time; assume many queries;

  assume static graph or require update operations!

AND / OR

- exploit special properties of the network (e.g., planar, hierarchical)

  fail when the given graph has not the desired properties!

$\leadsto$ not a solution for general graphs,

but can be very efficient for many practically relevant cases

Speedup Techniques

- require additional data (e.g., node coordinates)

AND / OR

- preprocess the graph and generate auxiliary data (e.g., ‘signposts’)

AND / OR

- exploit special properties of the network (e.g., planar, hierarchical)

Goals

- fast queries
- accurate results
- scale invariant / support all types of queries
- fast preprocessing / deal with large networks
- low space consumption
- fast update operations
- simple

Overview

Transit Node Routing
very fast queries
[DIMACS 06, ALENEX 07, Science 07]

HH Star
goal−directed
[DIMACS 06]

Highway Hierarchies
foundation
[ESA 05, ESA 06]

Hwy−Node Routing
allow edge weight changes
[WEA 07]

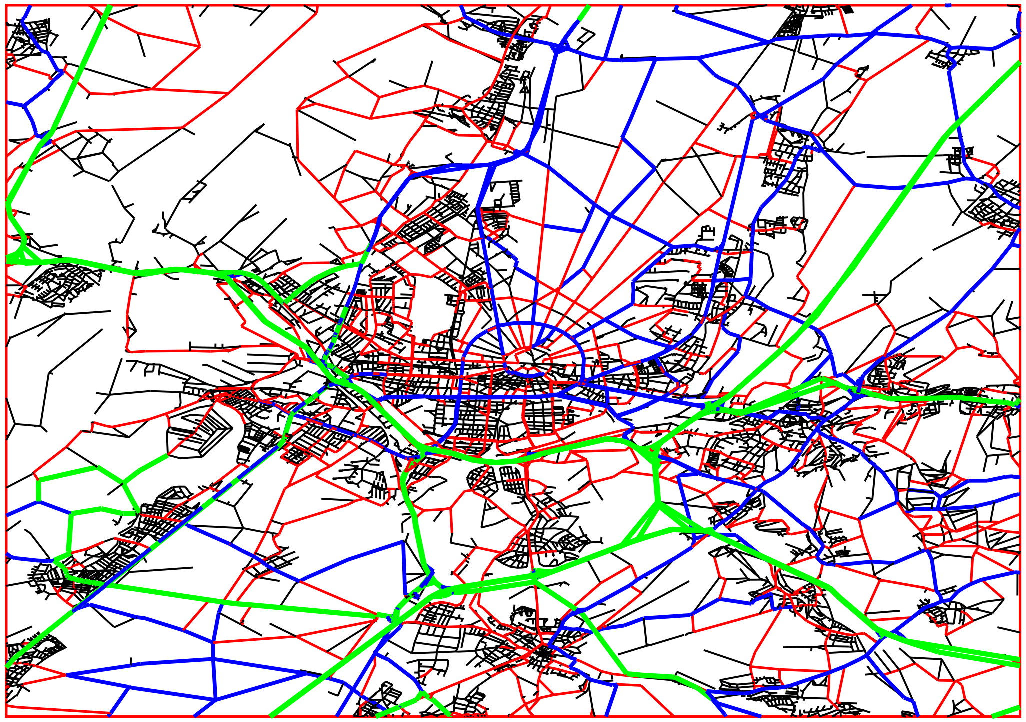
Many−to−Many
compute distance tables
[ALENEX 07]

Highway Hierarchies

[ESA 05, ESA 06]

Construction: iteratively alternate between

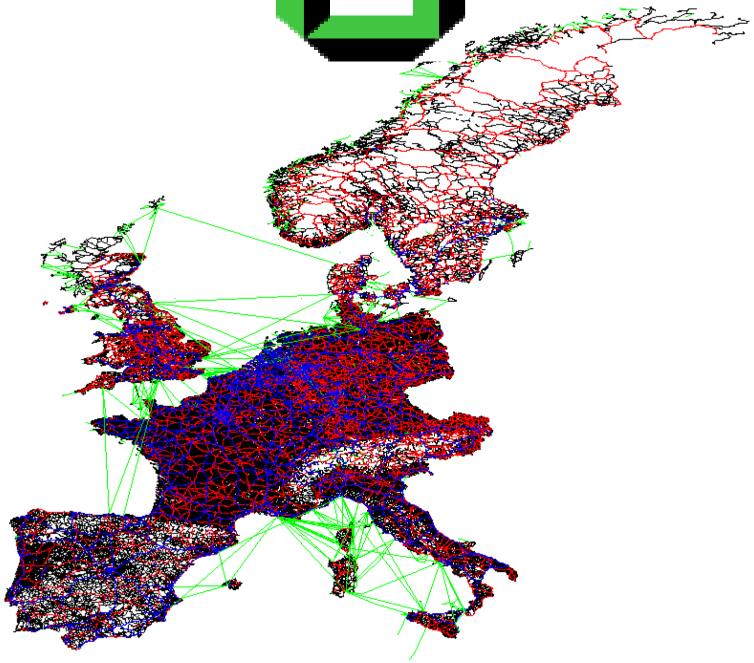
- removal of low degree nodes
- removal of edges that only appear on shortest paths close to source or target



yields a hierarchy of highway networks

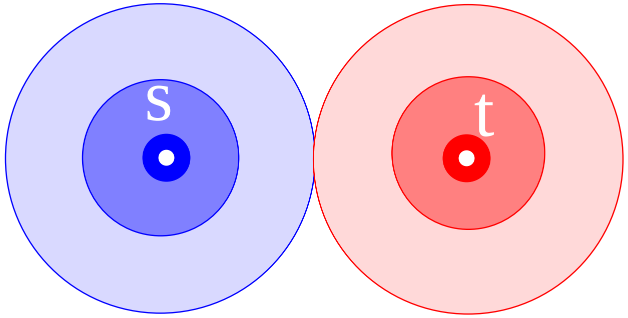
in a sense, classify roads / junctions by 'importance'

Highway Hierarchies



- foundation for our other methods
- directly allows point-to-point queries
- 13 min preprocessing
- 0.61 ms to determine the path length
- (0.80 ms to determine a complete path description)
- reasonable space consumption (48 bytes/node)

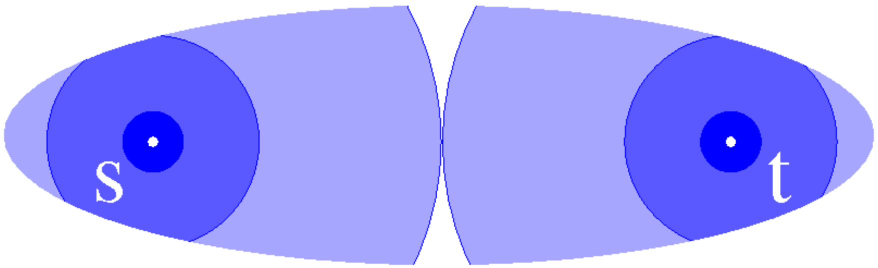
  can be reduced to 17 bytes/node



Highway Hierarchies Star

joint work with D. Delling, D. Wagner [DIMACS Challenge 06]

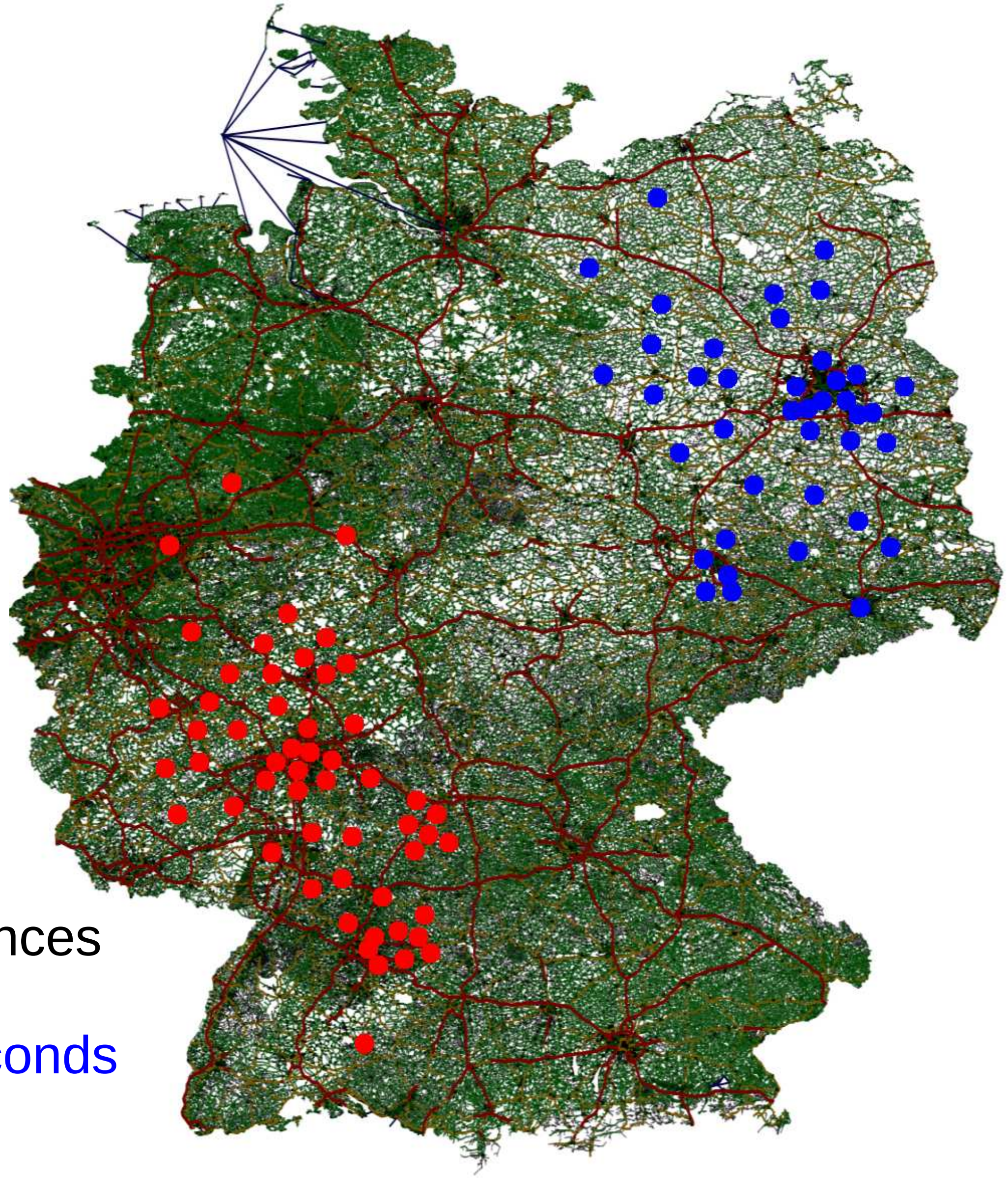
- combination of highway hierarchies with goal-directed search
- slightly reduced query times (0.49 ms)
- more effective
  - for approximate queries or
  - when a distance metric instead of a travel time metric is used



Many-to-Many Shortest Paths

joint work with S. Knopp, F. Schulz, D. Wagner

[ALENEX 07]



Given:

- graph $G = (V, E)$
- set of source nodes $S \subseteq V$
- set of target nodes $T \subseteq V$

Task: compute $|S| \times |T|$ distance table

containing the shortest path distances

- e.g., 10 000 $\times$ 10 000 table in 23 seconds

Transit-Node Routing

[DIMACS Challenge 06, ALENEX 07, Science 07]

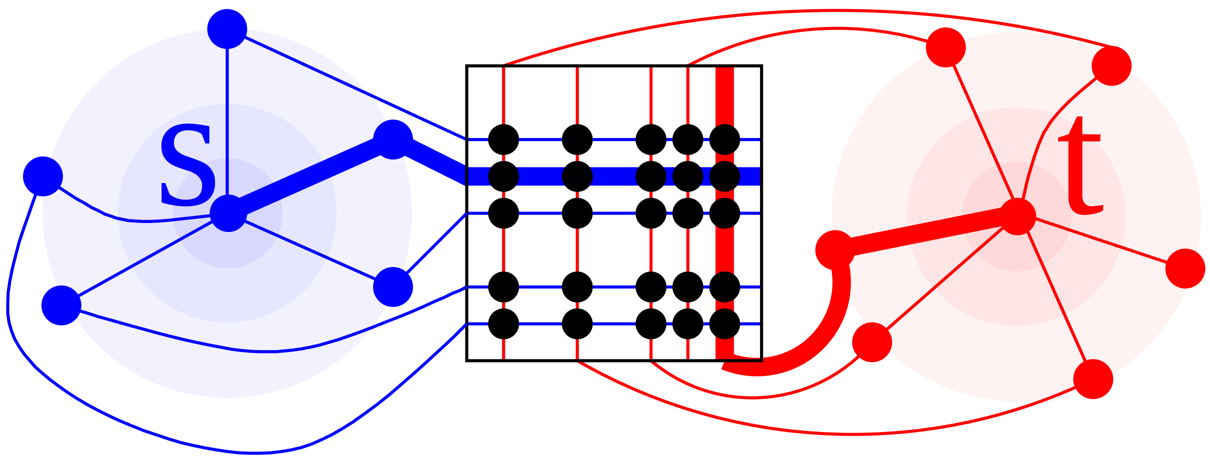
joint work with H. Bast, S. Funke, D. Matijevic

- *very* fast queries

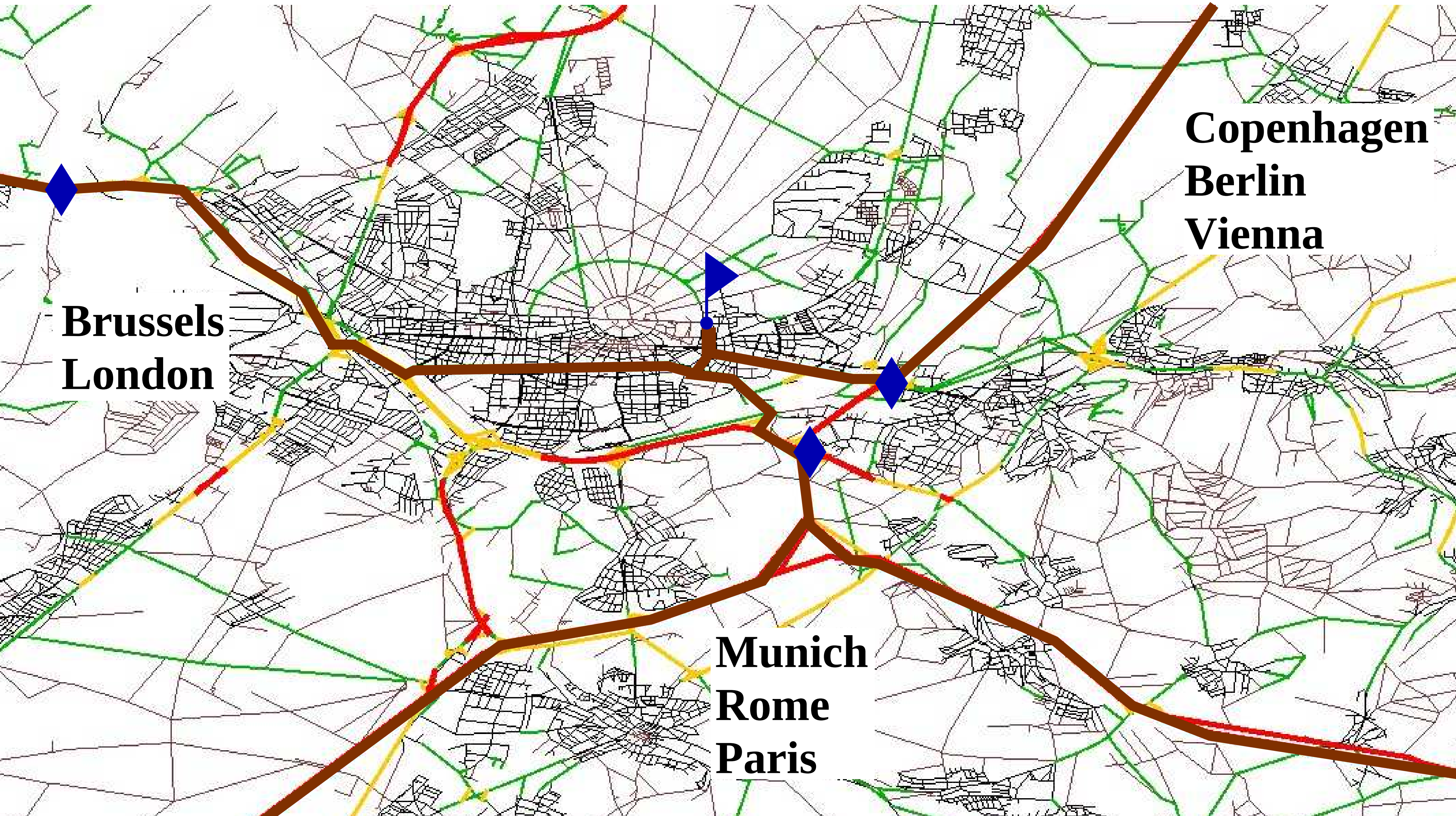
  (down to 4 μs, > 1 000 000 times faster than DIJKSTRA)

- winner of the 9th DIMACS Implementation Challenge

- more preprocessing time (1:15 h) and space (247 bytes/node) needed



Transit Node Routing



Transit-Node Routing

First Observation:

For long-distance travel: leave current location

via one of only a few ‘important’ traffic junctions,

called **access points** [in Europe $\approx$ 10]

($\rightsquigarrow$ we can afford to store all access points for each node)

Second Observation:

Each access point is relevant for several nodes. $\rightsquigarrow$

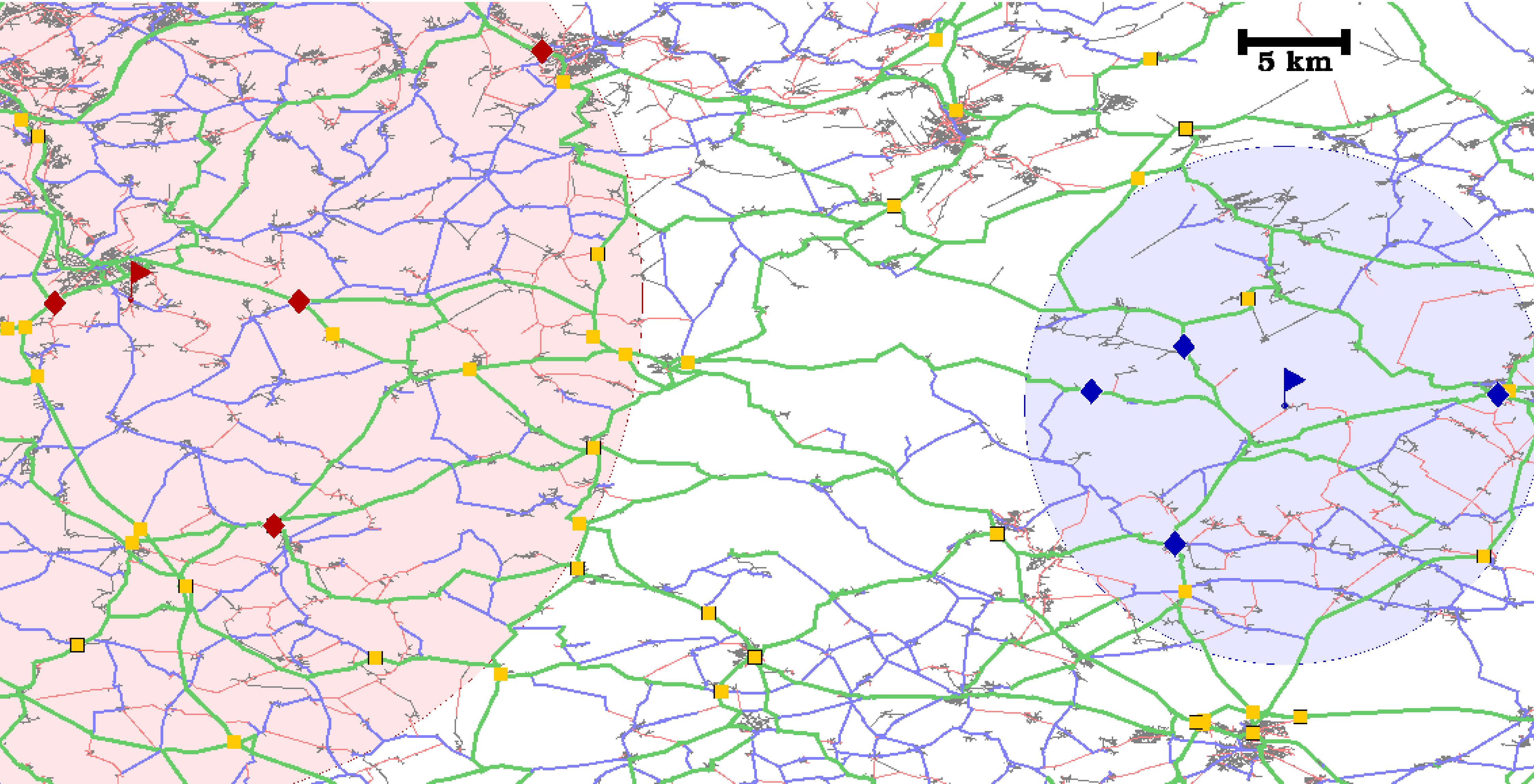
union of the access points of all nodes is small,

called **transit node set** [in Europe $\approx$ 10 000]

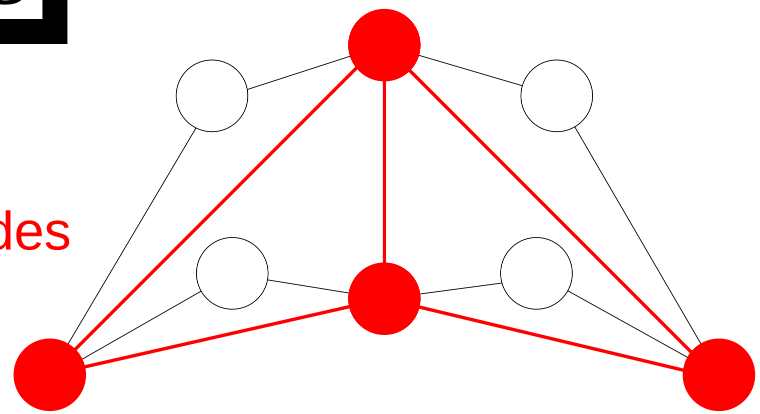
($\rightsquigarrow$ we can afford to store the distances between all transit node pairs)

Transit-Node Routing

Query: usually only a few table lookups



Highway-Node Routing

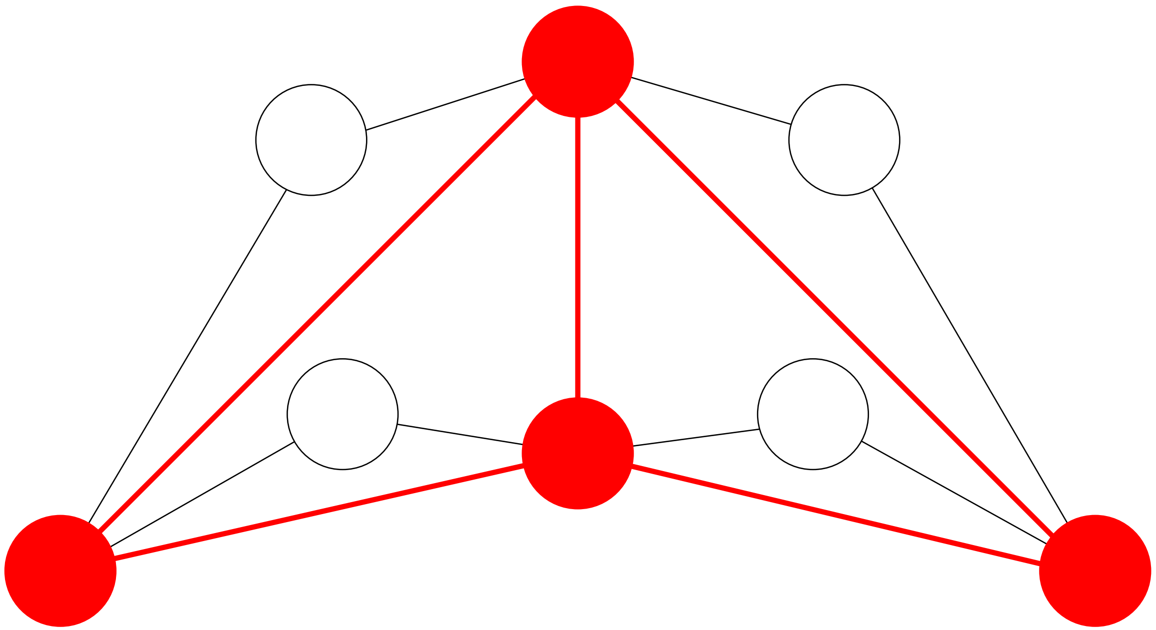


1. **basic concepts:** overlay graphs, covering nodes

2. lightweight, efficient **static approach**

3. **dynamic version**

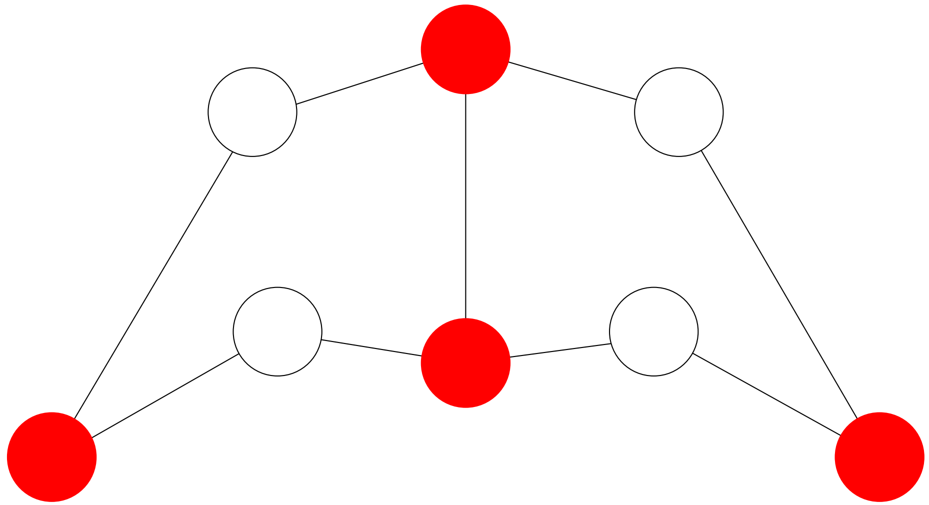
1. Basic Concepts



Overlay Graph: Definition

[Holzer, Schulz, Wagner, Weihe, Zaroliagis 2000–2007]

- graph $G = (V, E)$ is given
- select node subset $S \subseteq V$



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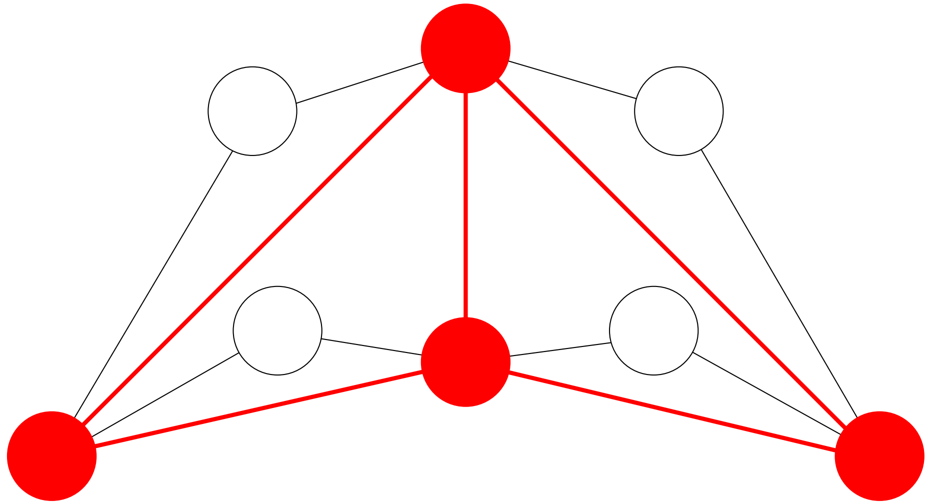


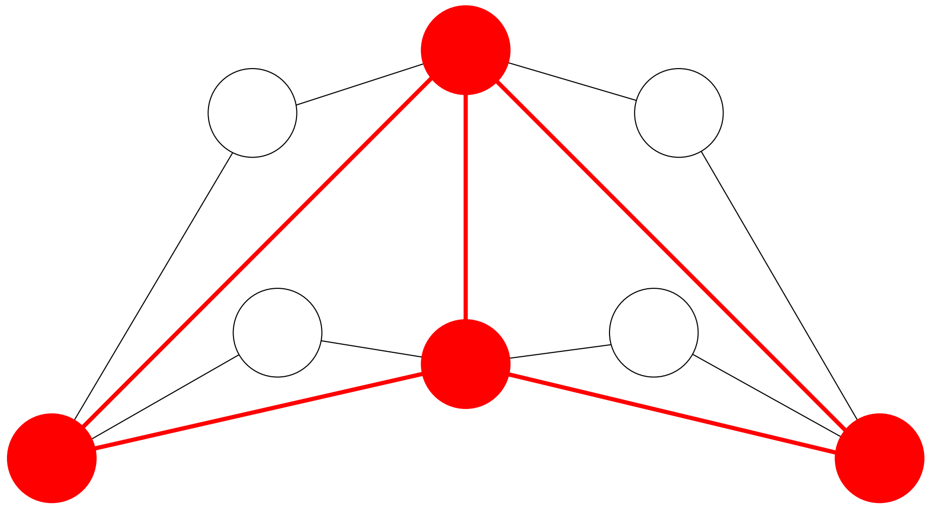
- overlay graph $G' := (S, E')$

  determine edge set E' s.t. shortest path distances are preserved

Minimal Overlay Graph

[Holzer, Schulz, Wagner, Weihe, Zaroliagis 2000–2007]

- graph $G = (V, E)$ is given
- select node subset $S \subseteq V$



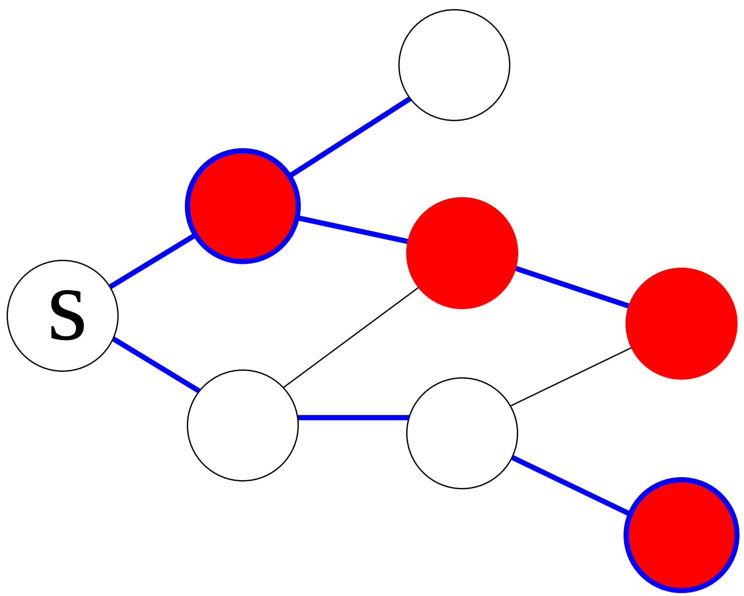
- minimal overlay graph $G' := (S, E')$ where

$E' := \{(s,t) \in S \times S \mid$ no inner node of the shortest s-t-path belongs to $S\}$

Covering Nodes

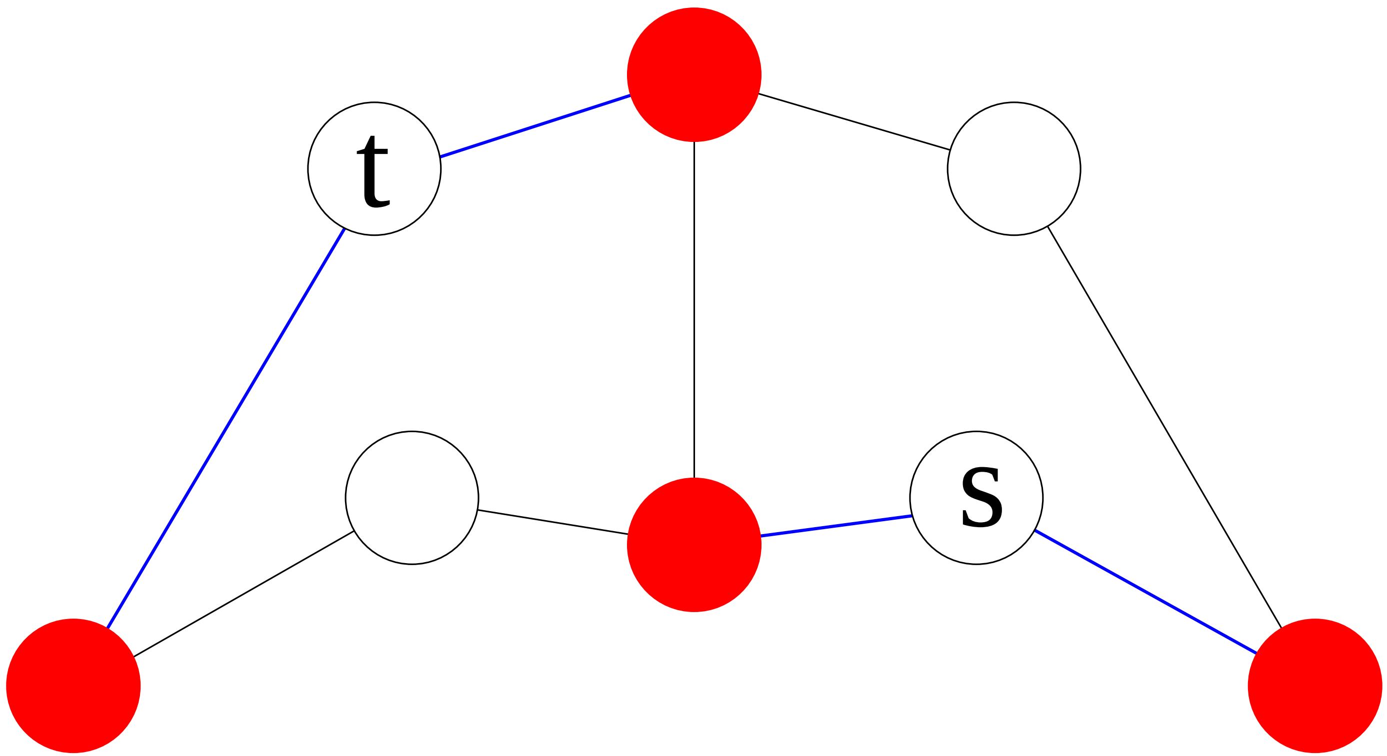
Definitions:

- covered branch: contains a node from S
- covered tree: all branches covered
- covering nodes: on each branch, the node $u \in S$ closest to the root s



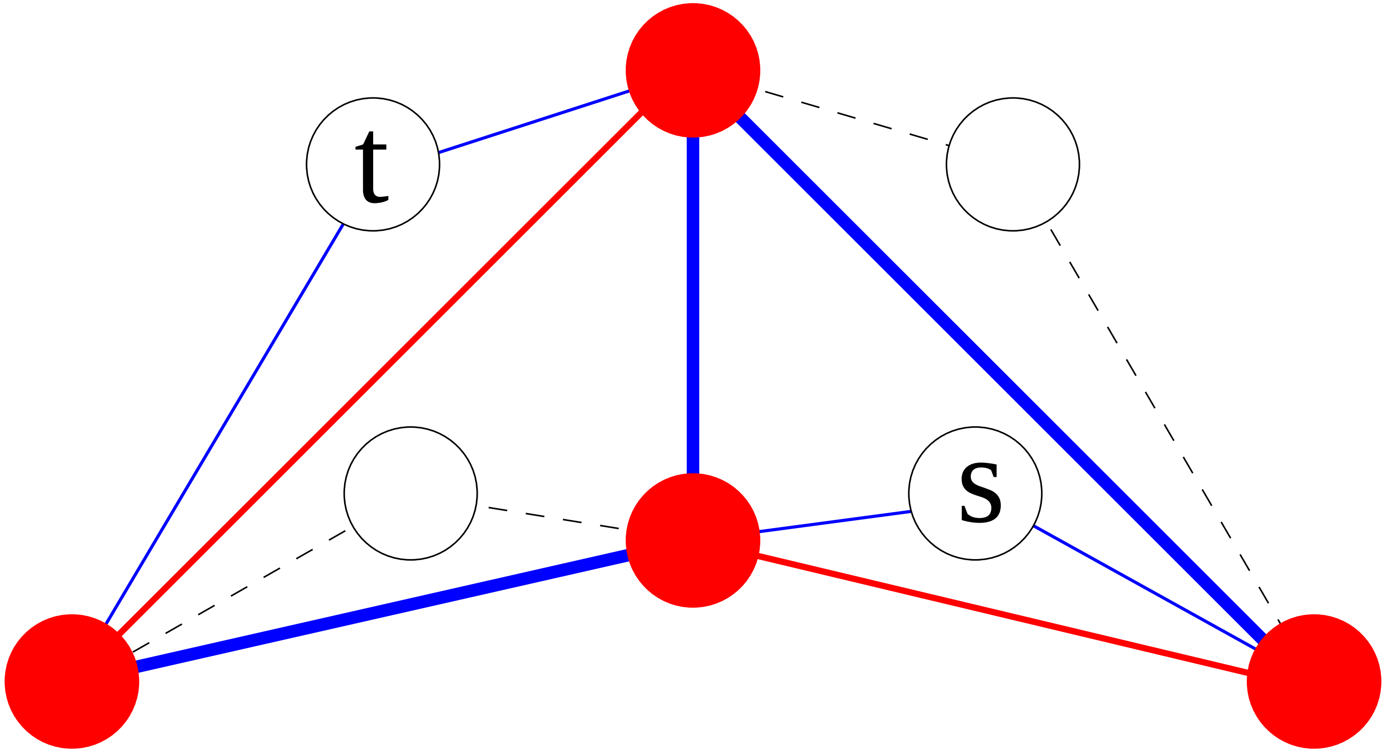
Query: Intuition

- bidirectional
- perform search in G till search trees are covered by nodes in S



Query: Intuition

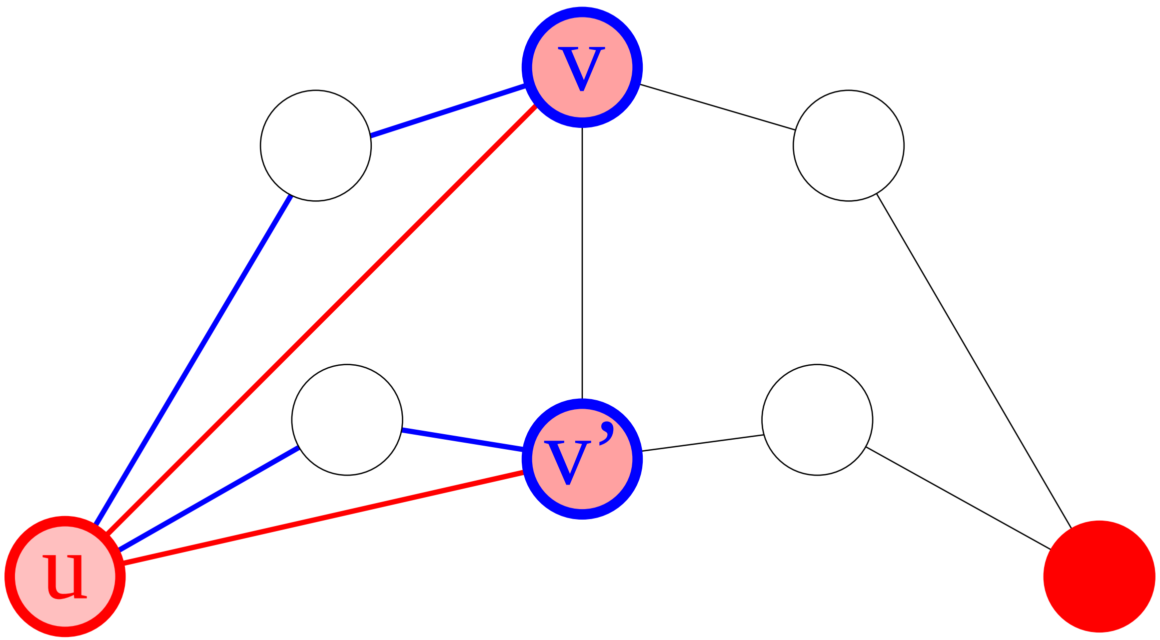
- bidirectional
- perform search in G till search trees are covered by nodes in S
- continue search only in G'



Overlay Graph: Construction

for each node $u \in S$

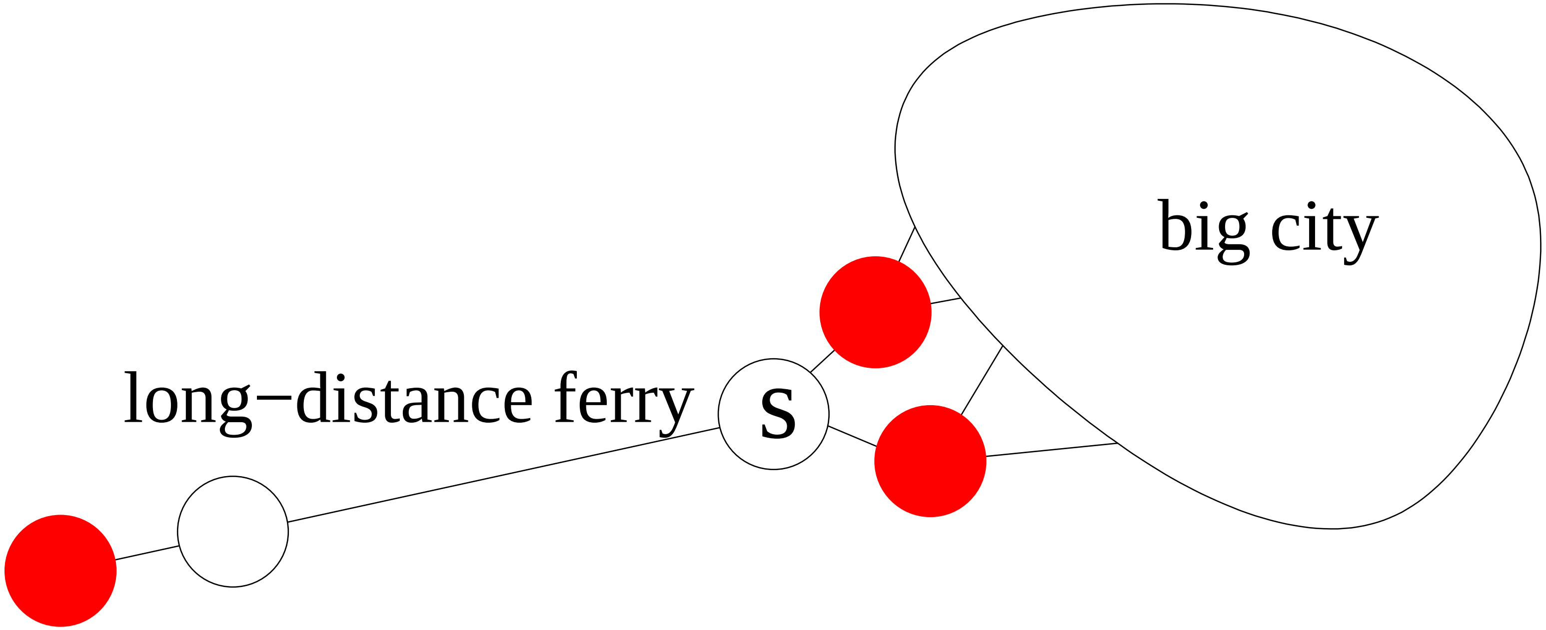
- perform a local search from u in G
- determine the covering nodes
- add an edge (u,v) to E' for each covering node v



Covering Nodes

Conservative Approach:

- stop searching in G when all branches are covered

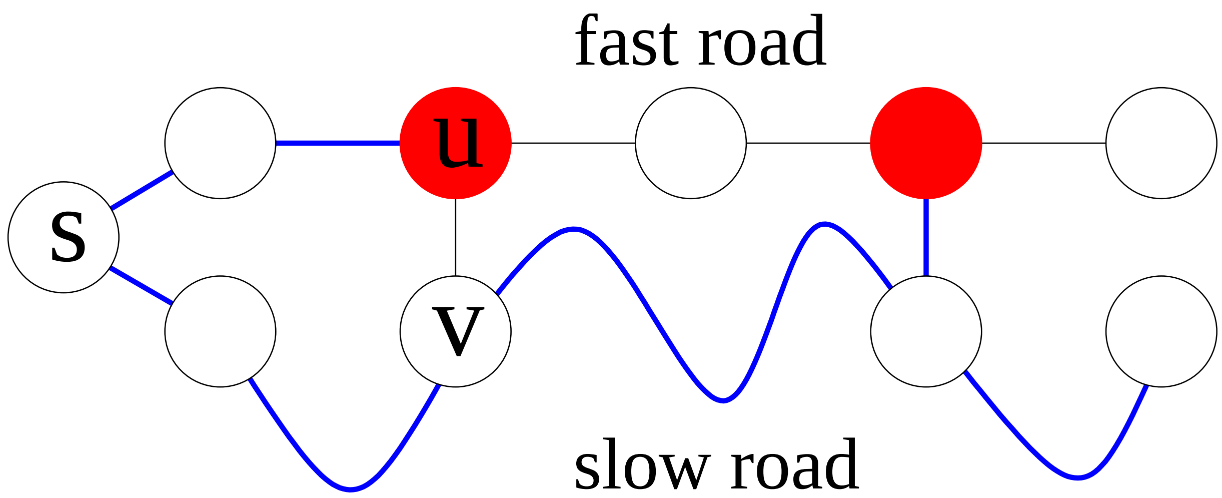


- can be very inefficient

Covering Nodes

Aggressive Approach:

- do not continue the search in G on covered branches



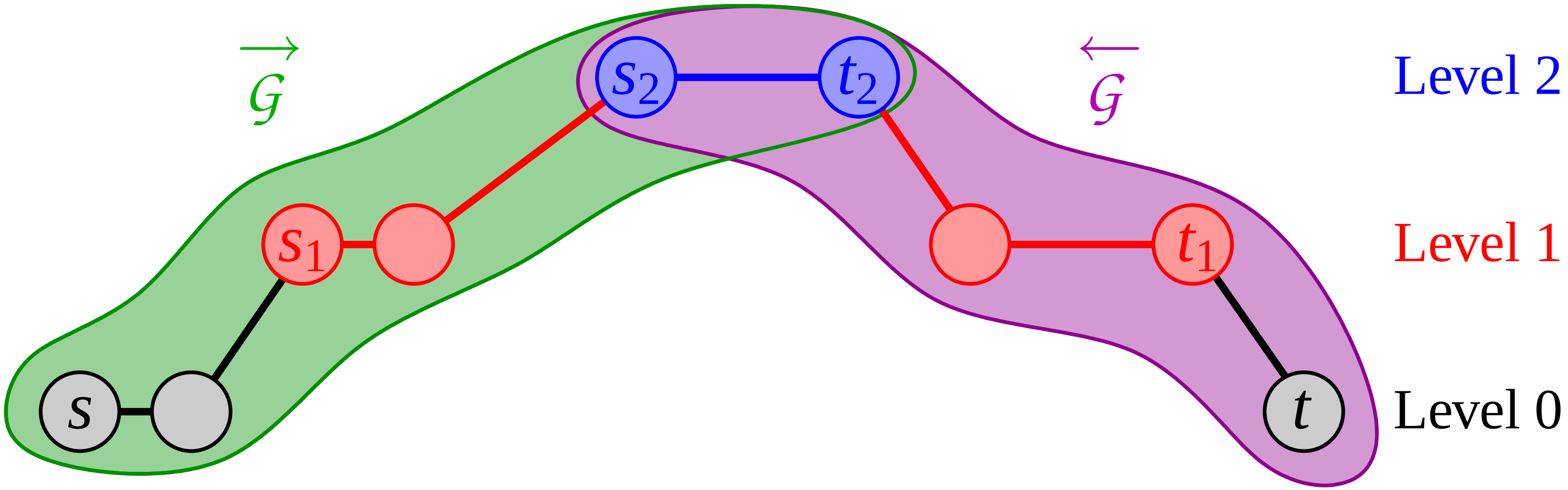
- can be very inefficient

Covering Nodes

Compromise:

- introduce parameter p
- do not continue the search in G on branches that already contain p nodes from S
- in addition: stop when all branches are covered
- $p = 1 \longrightarrow$ aggressive
- $p = \infty \longrightarrow$ conservative
- works very well in practice

2. Static Highway-Node Routing



Static Highway-Node Routing

- extend ideas from
  - multi-level overlay graphs [HolzerSchulzWagnerWeiheZaroliagis00–07]
  - highway hierarchies [SS05–06]
  - transit node routing [BastFunkeMatijevicSS06–07]
- use highway hierarchies to classify nodes by ‘importance’

  i.e., select node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots \supseteq S_L$

  (crucial distinction from previous separator-based approach)
- construct multi-level overlay graph

  $G_0 = G = (V,E), G_1 = (S_1,E_1), G_2 = (S_2,E_2), \ldots, G_L = (S_L,E_L)$

  (just iteratively construct overlay graphs)

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- extend ideas from
  - multi-level overlay graphs [HolzerSchulzWagnerWeiheZaroliagis00–07]
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- use highway hierarchies to classify nodes by 'importance'

  i.e., select node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots \supseteq S_L$ 13 min

  (crucial distinction from previous separator-based approach)
- construct multi-level overlay graph 2 min

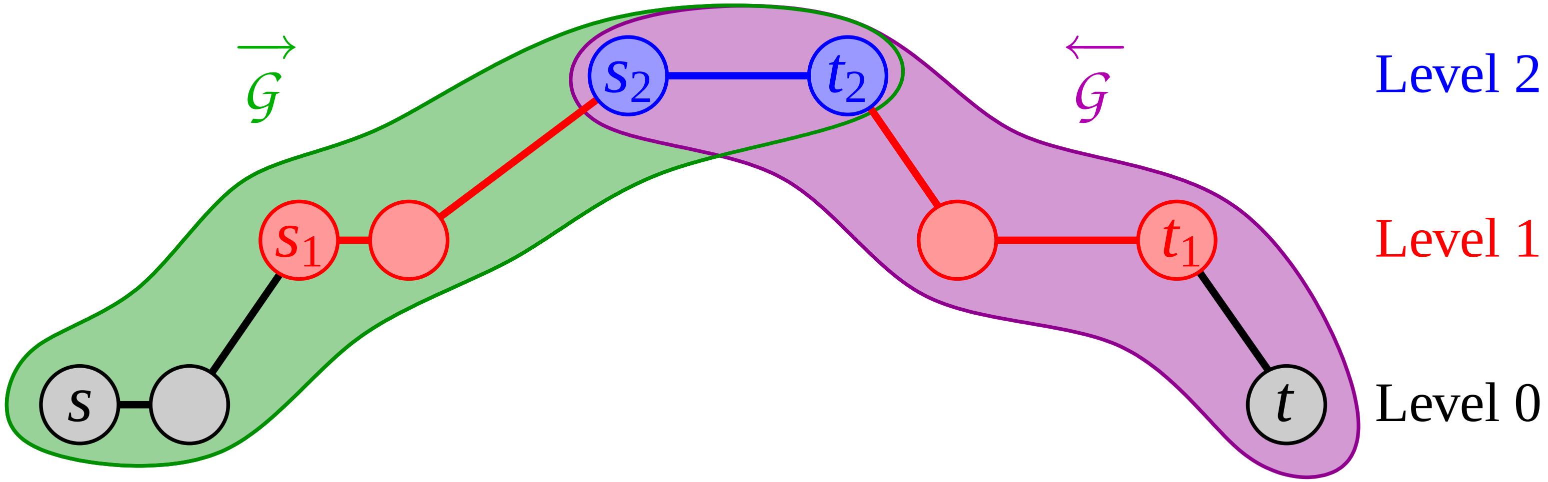
  $G_0 = G = (V,E), G_1 = (S_1,E_1), G_2 = (S_2,E_2), \ldots, G_L = (S_L,E_L)$

  (just iteratively construct overlay graphs)

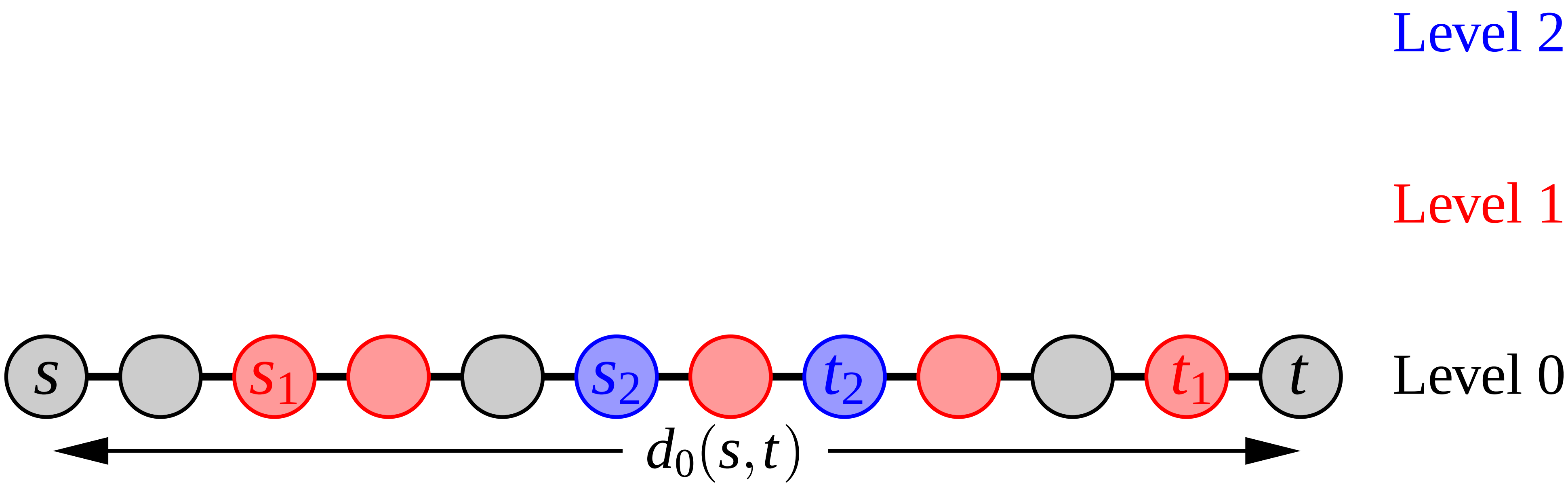
(experiments with a European road network with $\approx$ 18 million nodes)

Query: Aggressive Variant

- node level $\ell(u) := \max\{\ell \mid u \in S_\ell\}$
- forward search graph $\overrightarrow{\mathcal{G}} := \left(V, \left\{(u,v) \mid (u,v) \in \bigcup_{i=\ell(u)}^{L} E_i\right\}\right)$
- backward search graph $\overleftarrow{\mathcal{G}} := \left(V, \left\{(u,v) \mid (v,u) \in \bigcup_{i=\ell(u)}^{L} E_i\right\}\right)$
- perform one plain Dijkstra search in $\overrightarrow{\mathcal{G}}$ and one in $\overleftarrow{\mathcal{G}}$

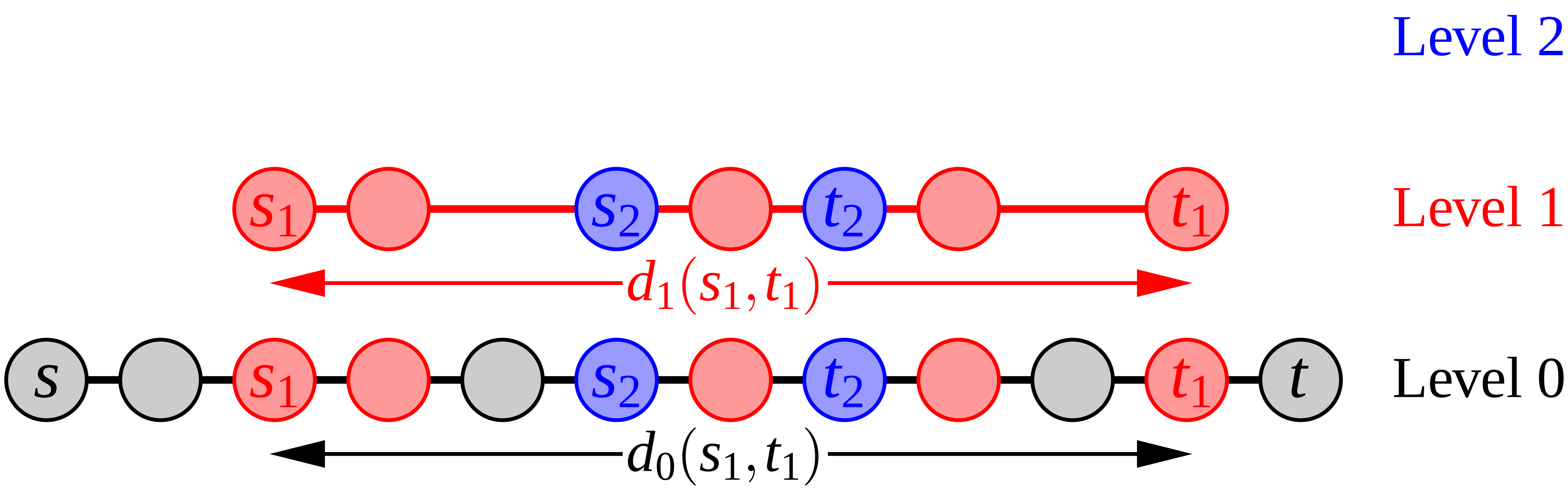


Proof of Correctness



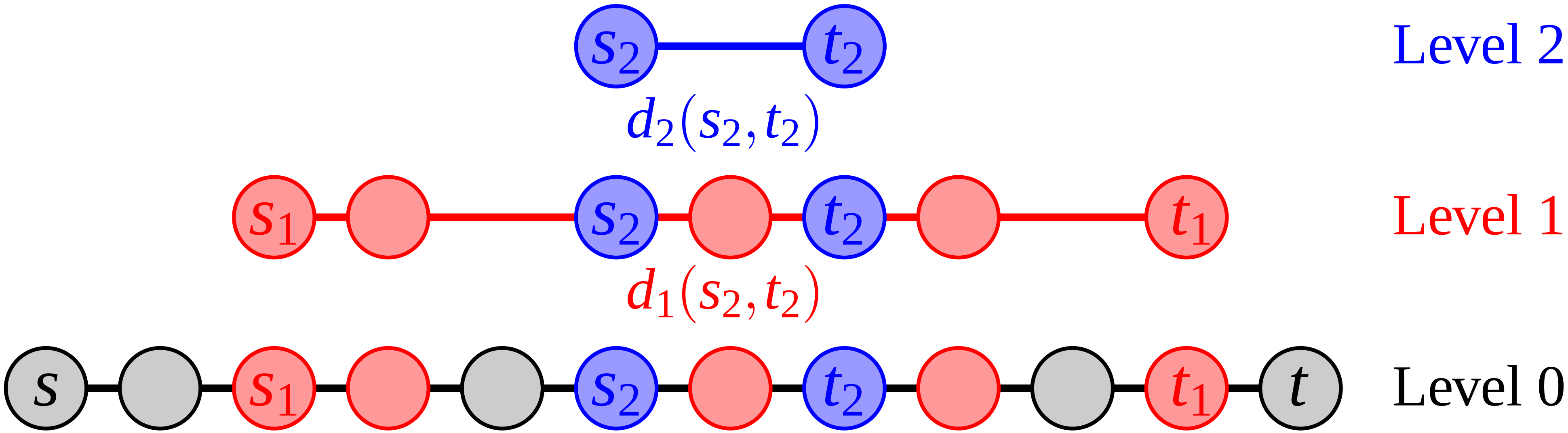
shortest path from s to t in $G = G_0$

Proof of Correctness



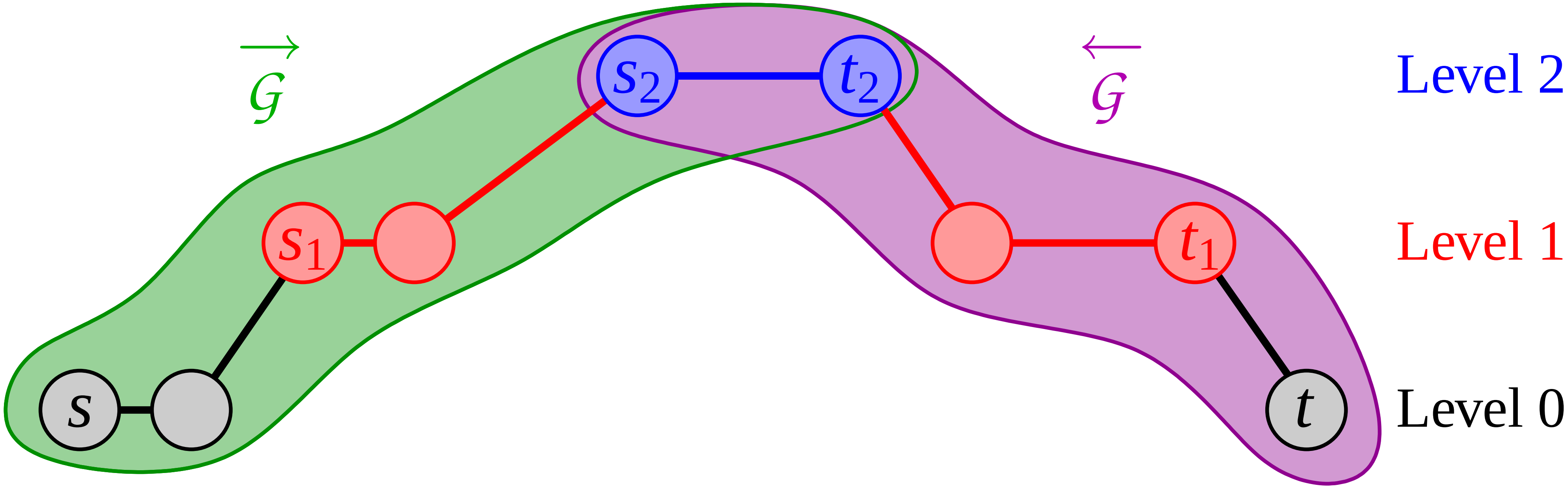
overlay graph G_1 preserves distance from $s_1 \in S_1$ to $t_1 \in S_1$

Proof of Correctness



overlay graph G_2 preserves distance from $s_2 \in S_2$ to $t_2 \in S_2$

Proof of Correctness

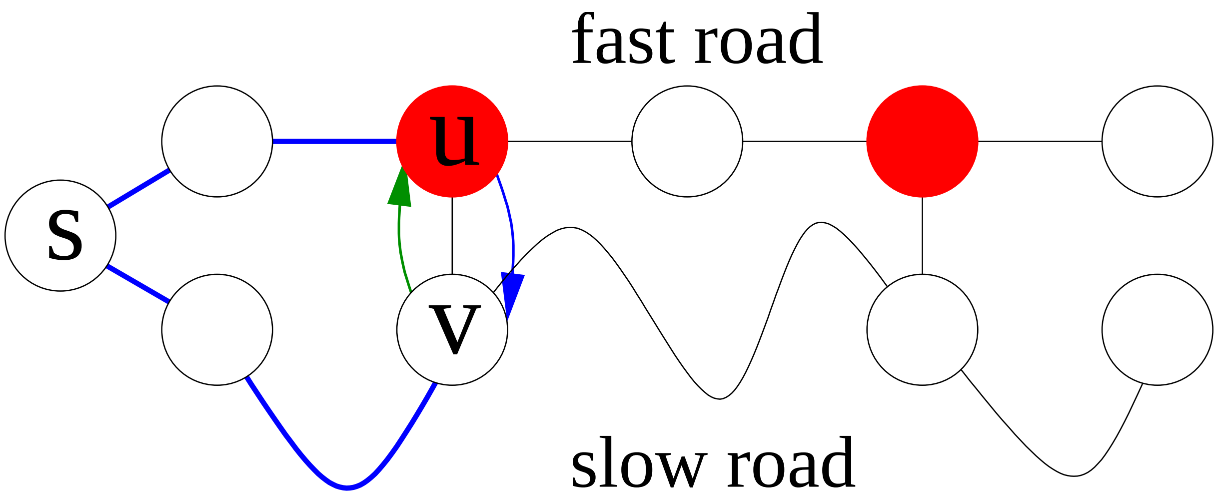


$$\overrightarrow{\mathcal{G}} := \left(V, \left\{(u,v) \mid (u,v) \in \bigcup_{i=\ell(u)}^{L} E_i\right\}\right)$$
$$\overleftarrow{\mathcal{G}} := \left(V, \left\{(u,v) \mid (v,u) \in \bigcup_{i=\ell(u)}^{L} E_i\right\}\right)$$

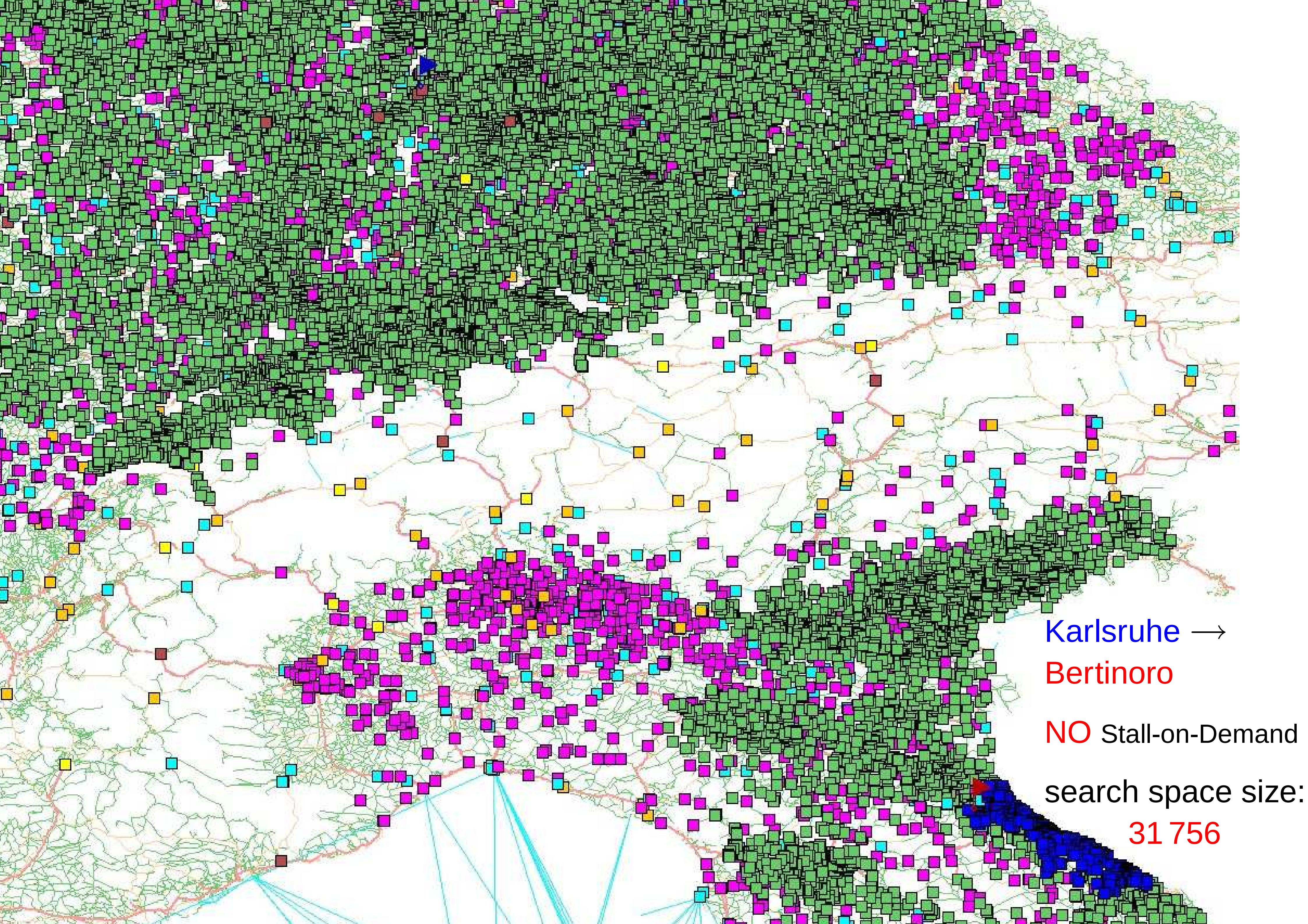
Stall-on-Demand

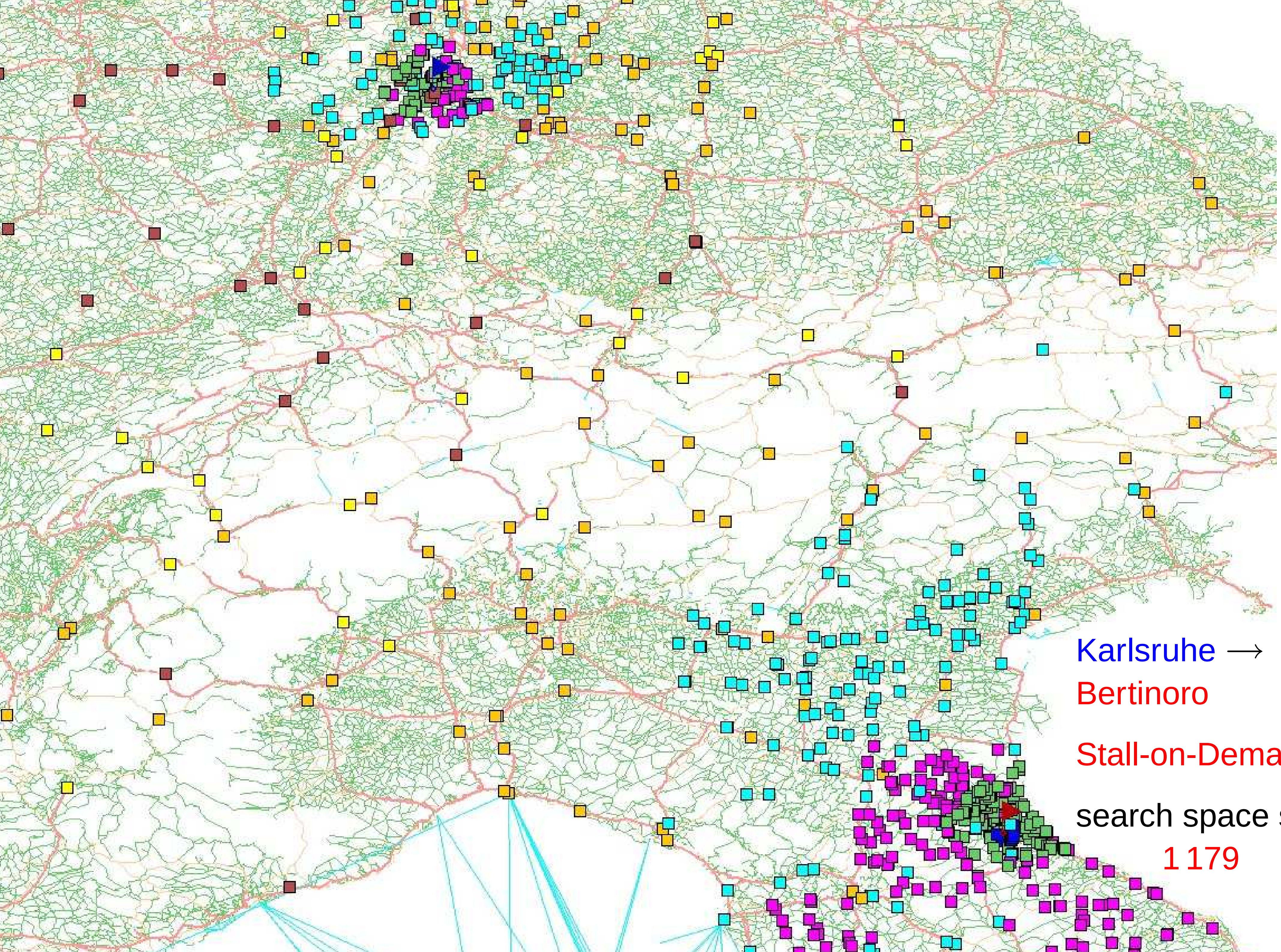
- a node v can 'wake' an already settled node u
- u can 'stall' v (if $\delta(u) + w(u,v) < \delta(v)$)

  i.e., search is not continued from v



- stalling can propagate to adjacent nodes
- does not invalidate correctness (only suboptimal paths are stalled)


Karlsruhe →
Bertinoro
NO Stall-on-Demand
search space size:
31 756



Karlsruhe →

Bertinoro

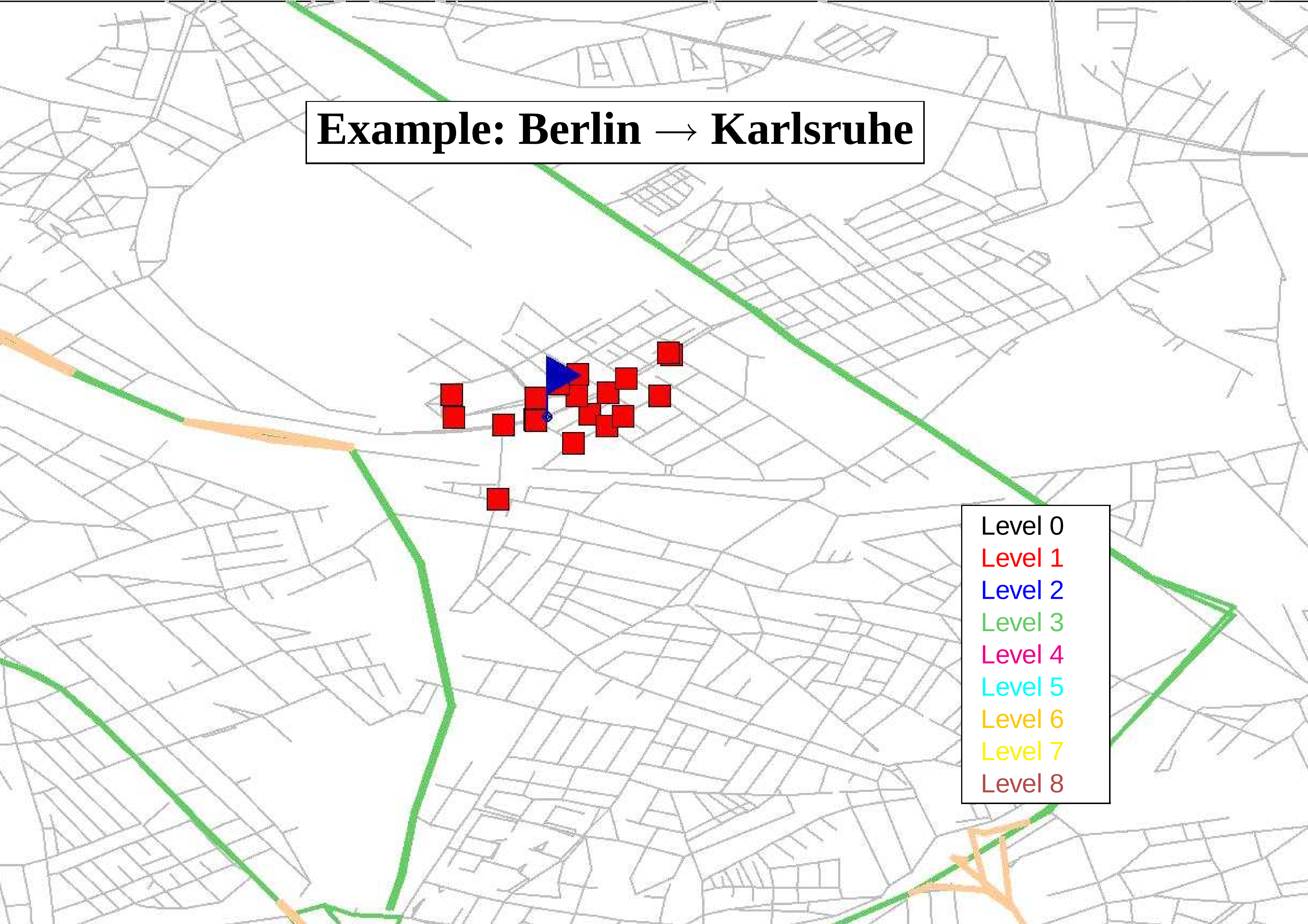
Stall-on-Demand

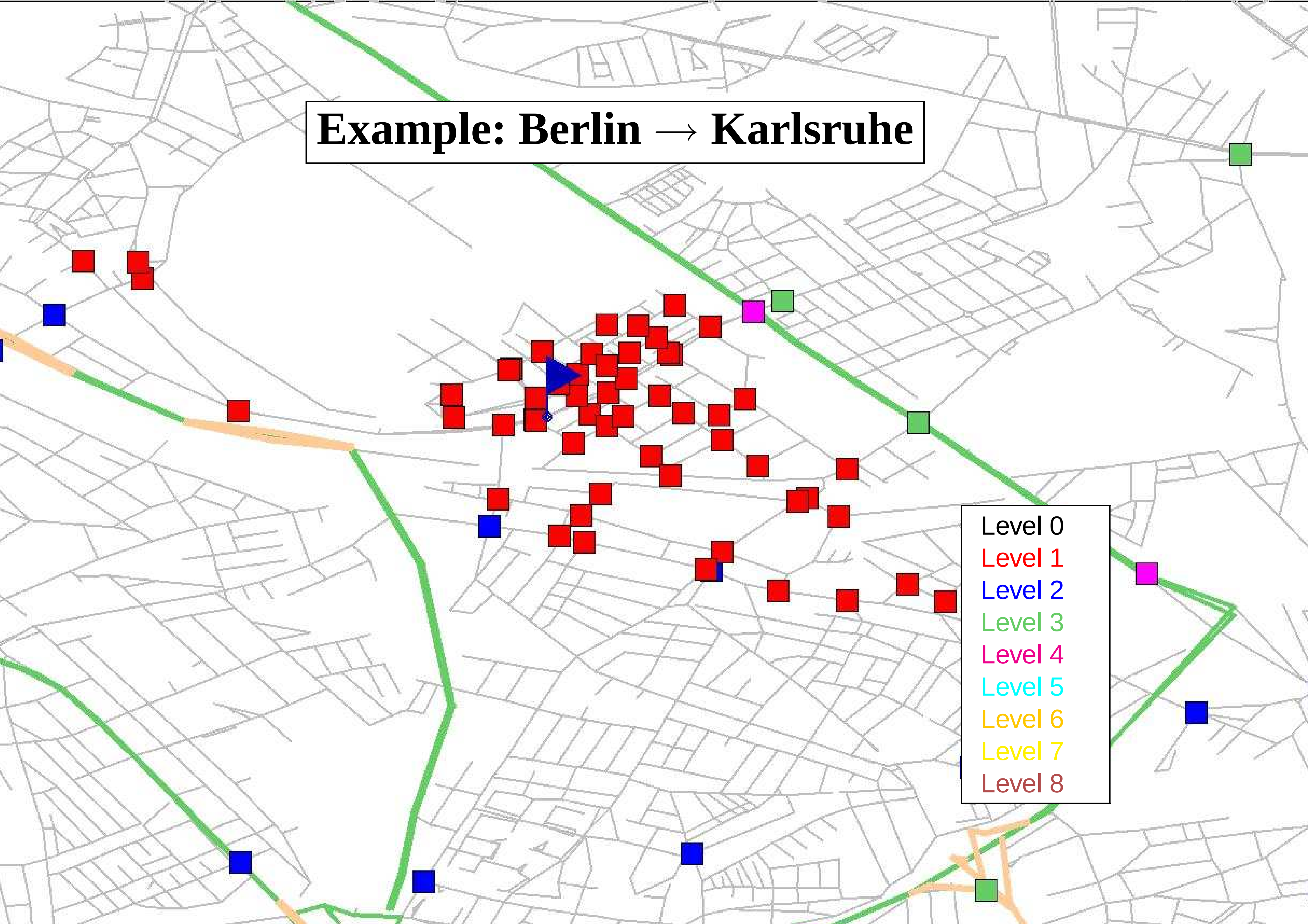
search space size:

1 179

Stall-on-Demand

```
const NodeID index = isReached(searchID, v);
if (edge.isDirected(1-dir) && index) {
  const PQData& data = pqData(searchID, index);
  EdgeWeight vKey = data.stalled() ? data.stallKey() : pqKey(searchID,index);
  if (vKey + edge.weight() < parentDist) {
    pqData(searchID, parent.index).stallKey(vKey + edge.weight());
    queue< pair<NodeID, EdgeWeight> > _stallQueue;
    _stallQueue.push(pair<NodeID,EdgeWeight>(parent.nodeID,vKey+edge.weight()));
    while (! _stallQueue.empty()) {
      u = _stallQueue.front().first;
      key = _stallQueue.front().second;
      _stallQueue.pop();
      for (EdgeID e = _graph->firstEdge(u); e < _graph->lastEdge(u); e++) {
        const Edge& edge = _graph->edge(e);
        if (! edge.isDirected(searchID)) continue;
        NodeID index = isReached(searchID, edge.target());
        if (index) {
          const EdgeWeight newKey = key + edge.weight();
          if (newKey < pqKey(searchID, index)) {
            PQData& data = pqData(searchID, index);
            if (! data.stalled()) {
              data.stallKey(newKey);
              _stallQueue.push(pair<NodeID,EdgeWeight>(edge.target(), newKey));
    } } } } }
    return;
} }
```


Example: Berlin → Karlsruhe
Level 0
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7
Level 8



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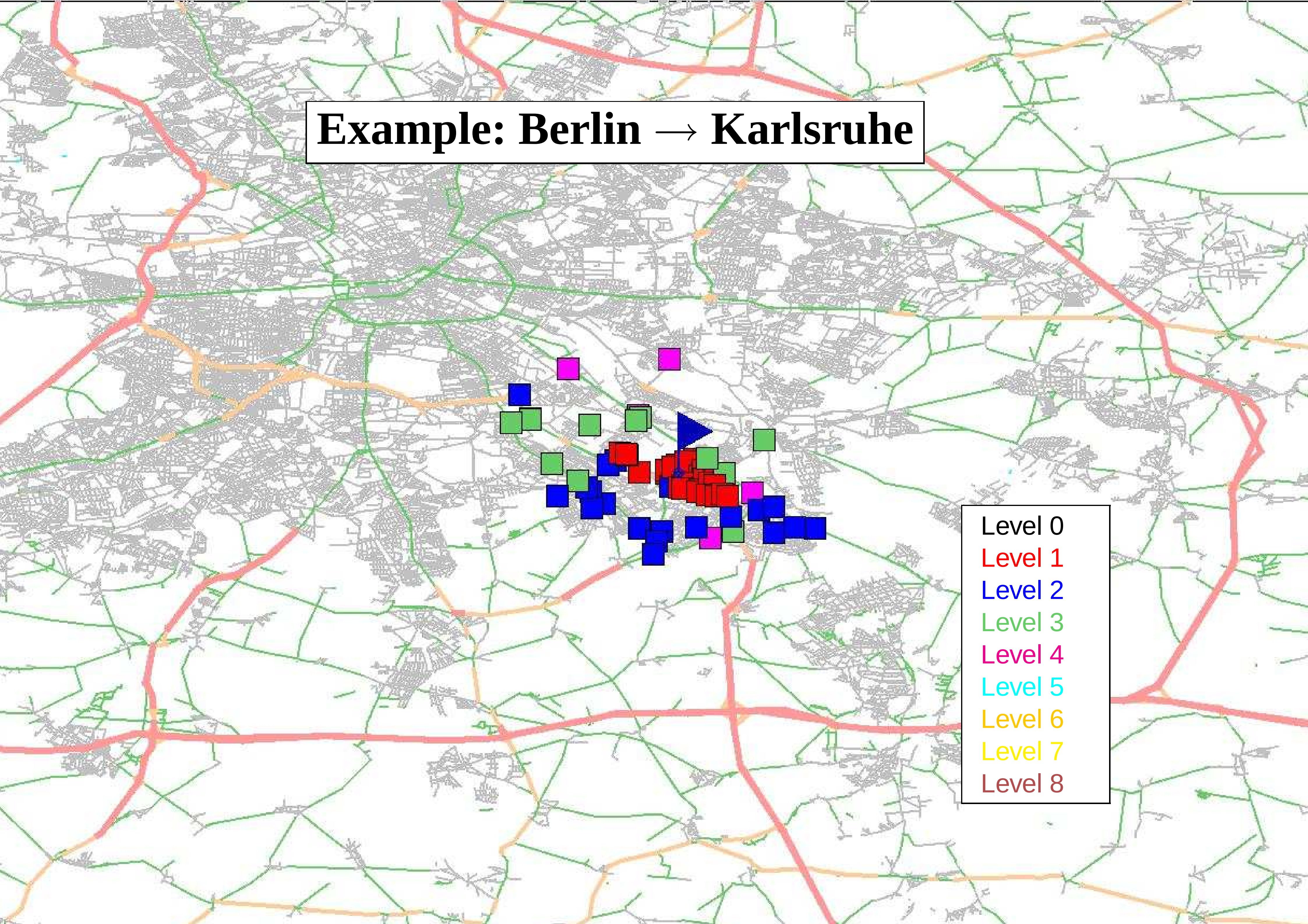

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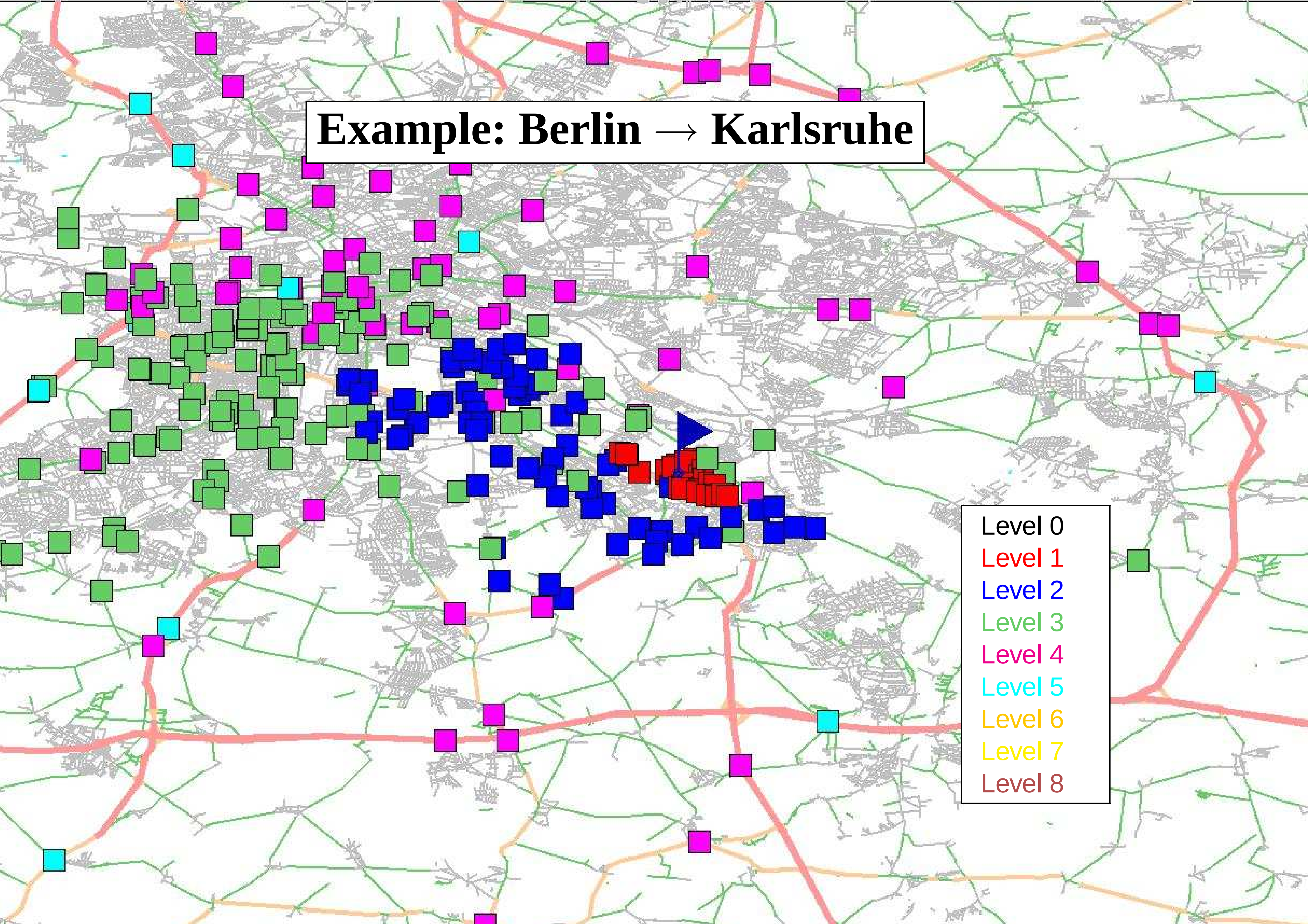

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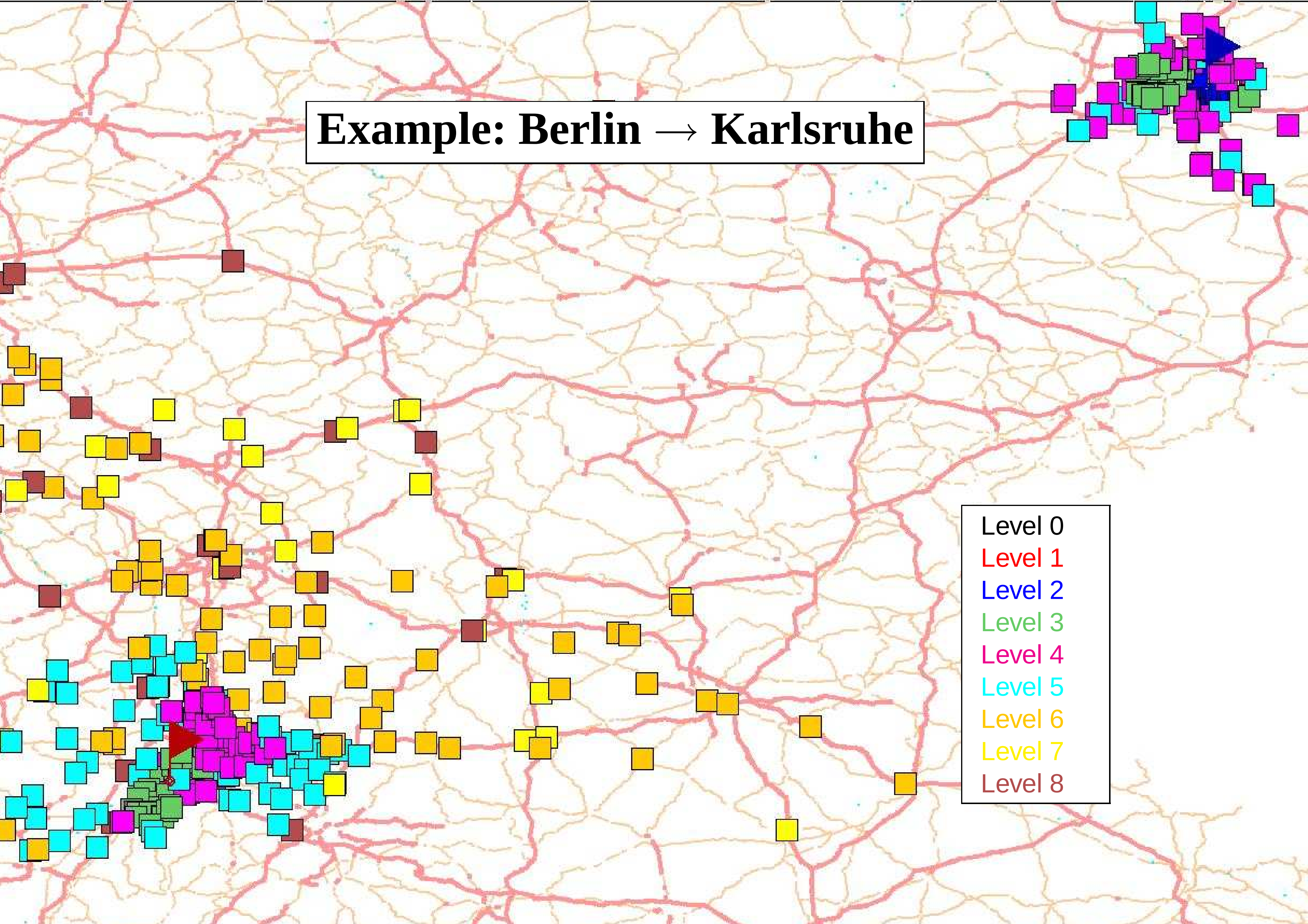

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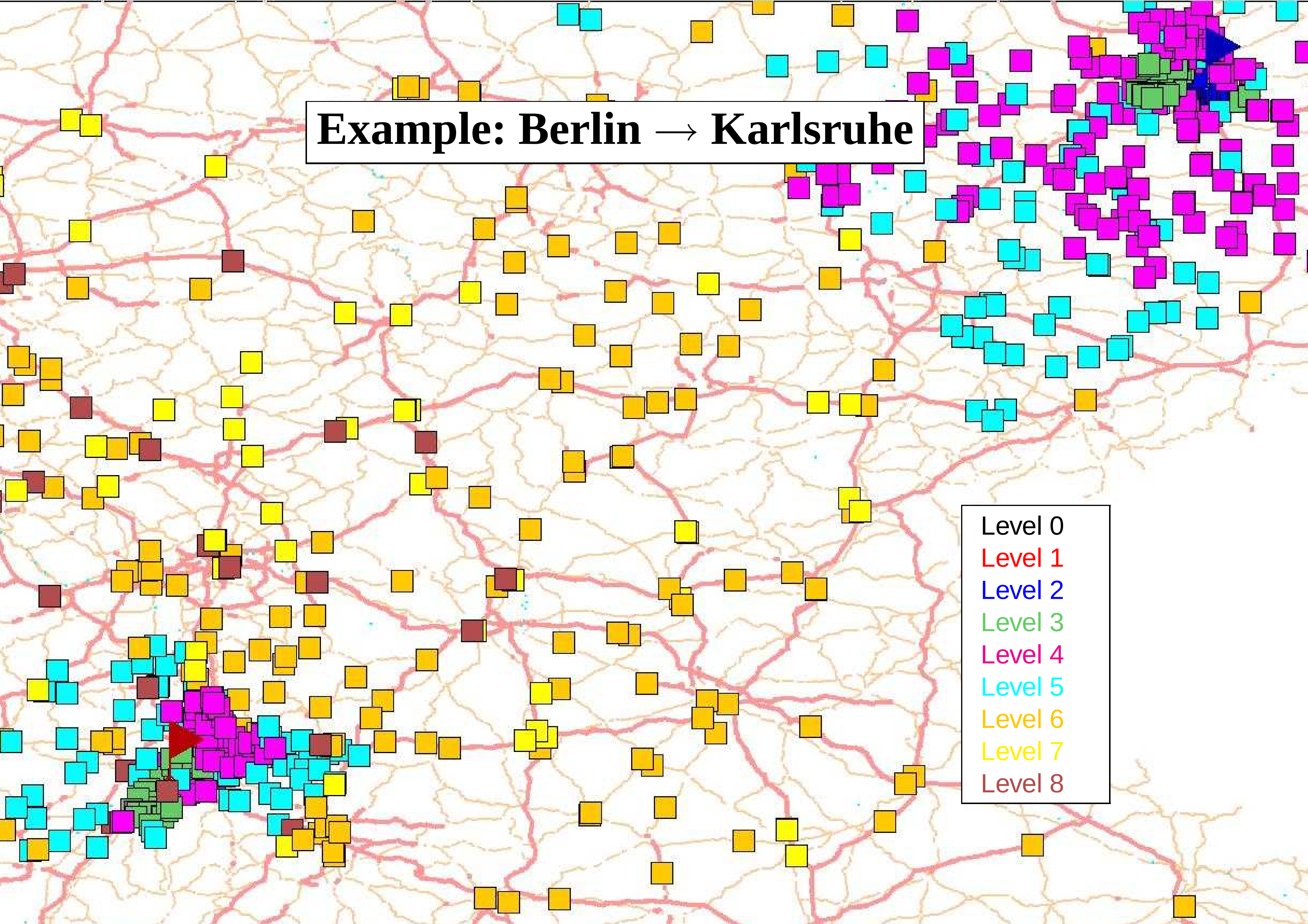
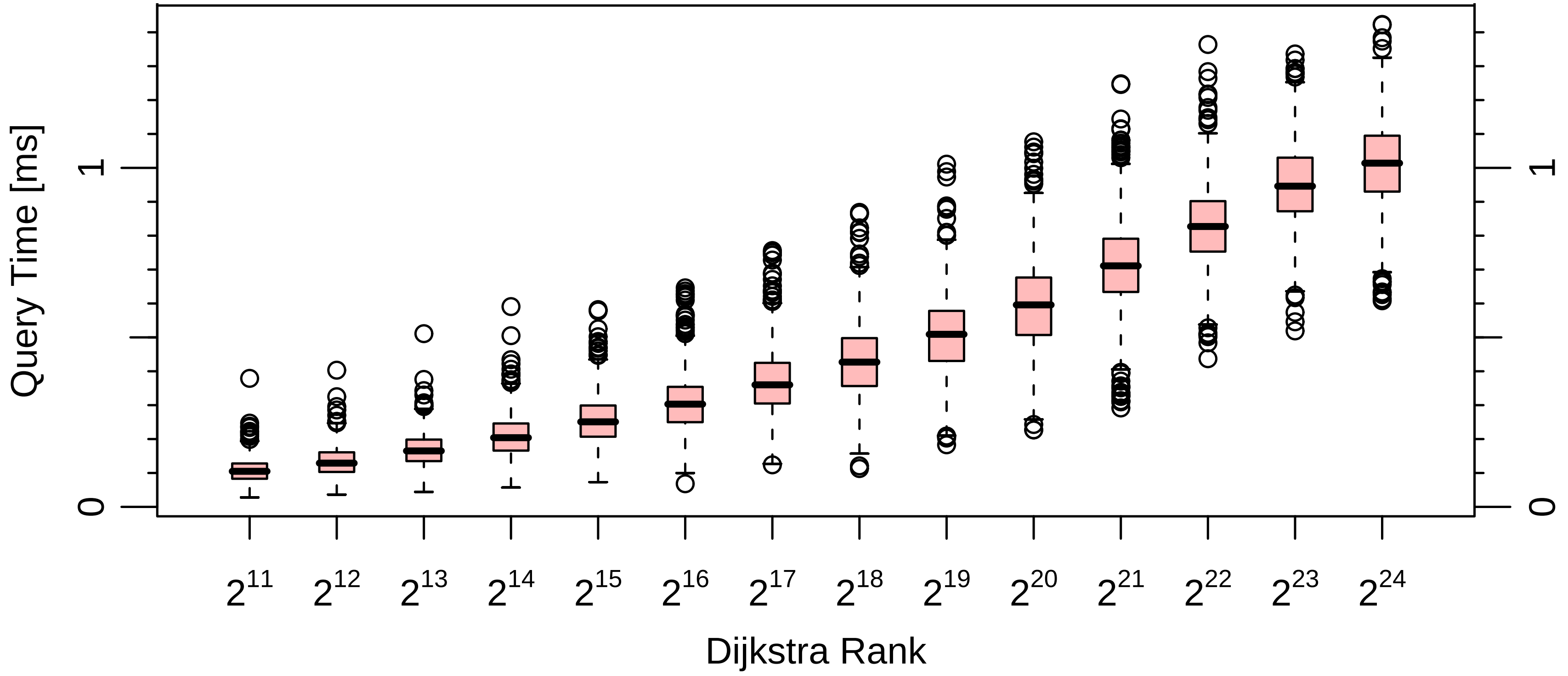
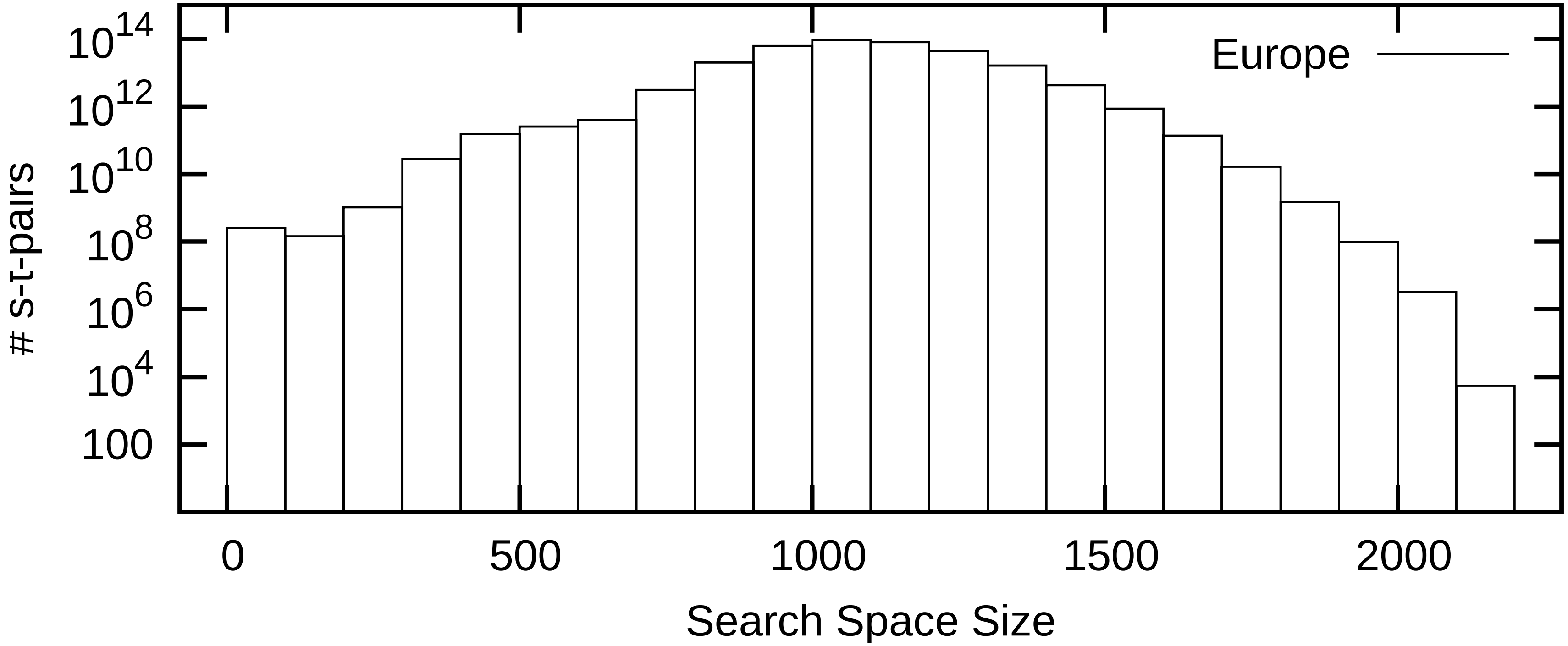


Local Queries



Per-Instance Worst-Case Guarantee



max = 2 148 nodes

Memory Consumption / Query Time

different trade-offs between memory consumption and query time

for example:

- 9.5 bytes per node overhead $\longrightarrow$ 0.89 ms

  store complete multi-level overlay graph

- 0.7 bytes per node overhead $\longrightarrow$ 1.44 ms

  store only forward and backward search graph $\overrightarrow{\mathcal{G}}$ and $\overleftarrow{\mathcal{G}}$

  ($\overrightarrow{\mathcal{G}}$ and $\overleftarrow{\mathcal{G}}$ are independent of s and t)

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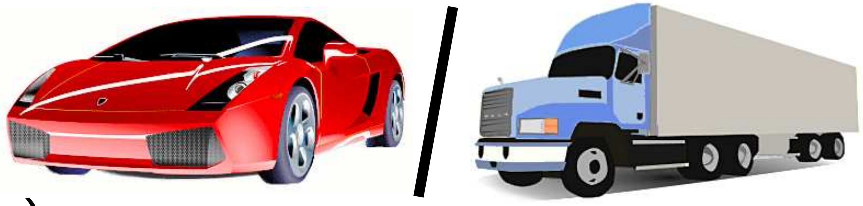
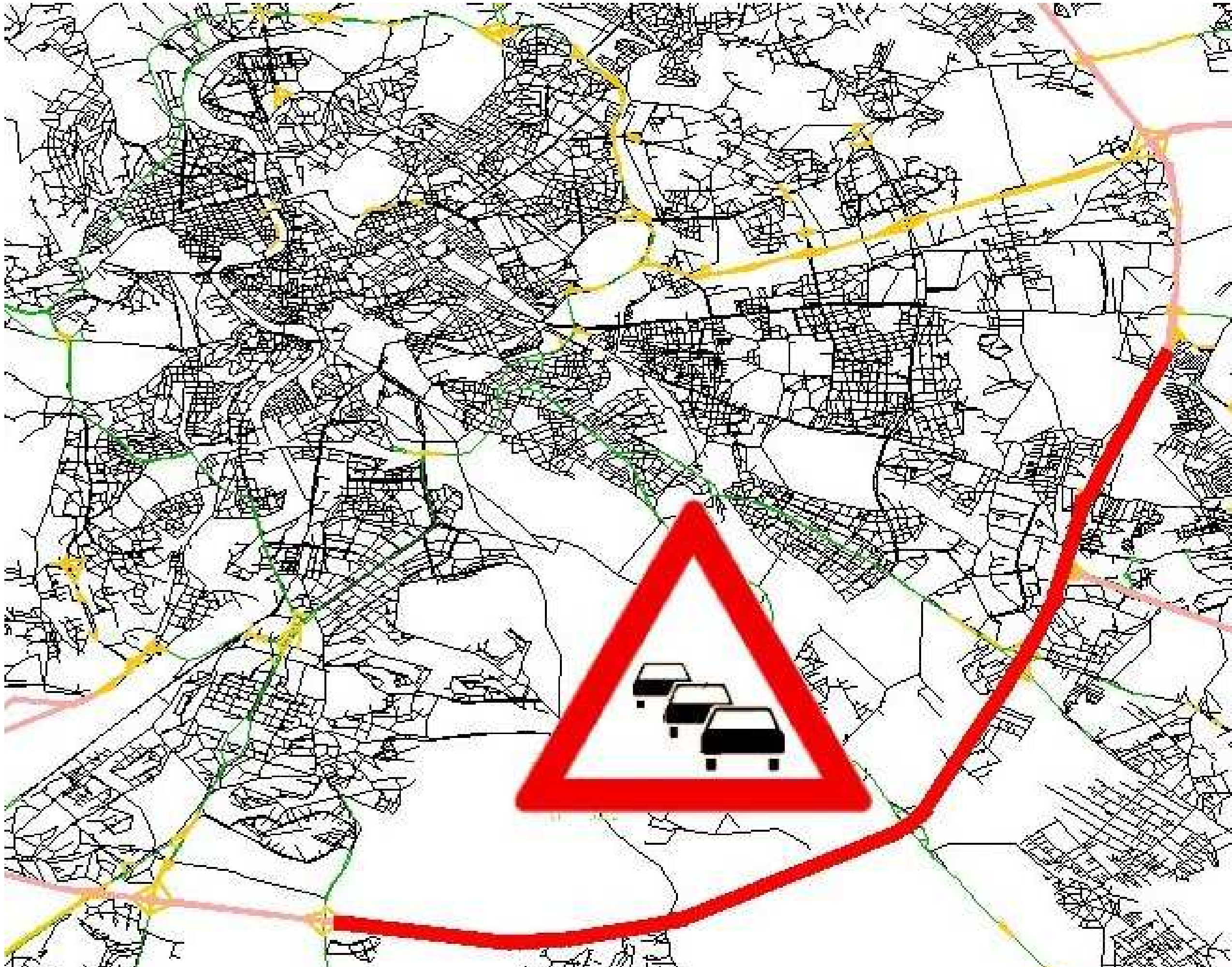
numbers refer to the Western European road network with 18 million nodes

3. Dynamic Highway-Node Routing

Dynamic Scenarios



- change entire cost function
  (e.g., use different speed profile)



- change a few edge weights
  (e.g., due to a traffic jam)

Constancy of Structure

Assumption:

- structure of road network does not change

  (no new roads, road removal = set weight to ∞)

  $\rightsquigarrow$ not a significant restriction

- classification of nodes by 'importance' might be slightly perturbed, but not completely changed

  (e.g., a sports car and a truck both prefer motorways)

  $\rightsquigarrow$ performance of our approach relies on that

  (not the correctness)

Dynamic Highway-Node Routing

change entire cost function

- keep the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$
- recompute the overlay graphs

speed profile	default	fast car	slow car	slow truck	distance
constr. [min]	1:40	1:41	1:39	1:36	3:56
query [ms]	1.17	1.20	1.28	1.50	35.62
#settled nodes	1 414	1 444	1 507	1 667	7 057

Dynamic Highway-Node Routing

change a few edge weights

- **server scenario:** if something changes,
  - update the preprocessed data structures
  - answer many subsequent queries very fast

- **mobile scenario:** if something changes,
  - it does not pay to update the data structures
  - perform single ‘prudent’ query that
    takes changed situation into account

Dynamic Highway-Node Routing

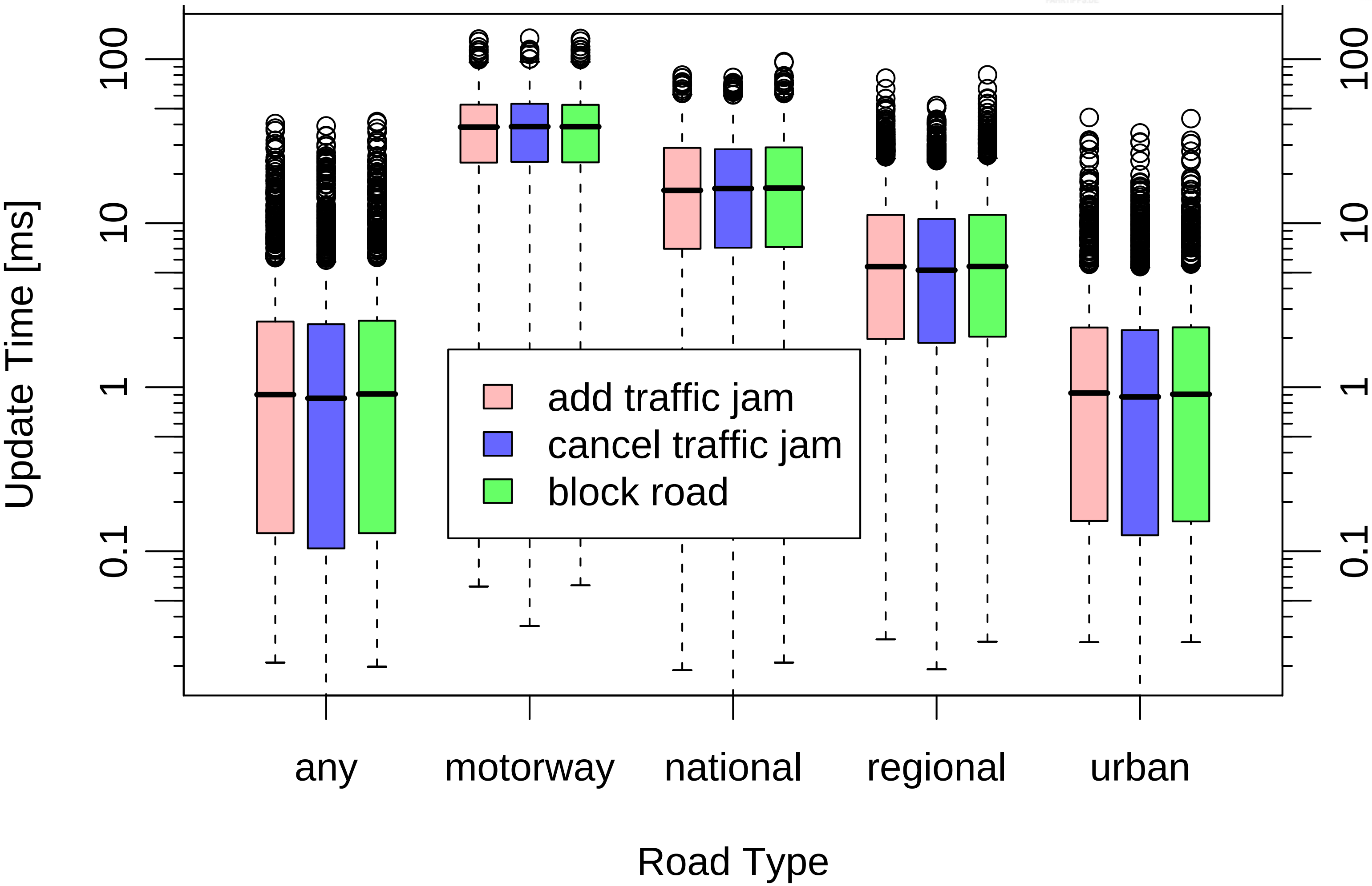
change a few edge weights, server scenario

- **keep** the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$

- **recompute** only **possibly affected parts** of the overlay graphs
  - the computation of the level-ℓ overlay graph consists of $|S_\ell|$ local searches to determine the respective covering nodes
  - if the initial local search from $v \in S_\ell$ has not touched a now modified edge (u,x), that local search need not be repeated
  - we manage sets $A_u^\ell = \{v \in S_\ell \mid v\text{'s level-}\ell \text{ preprocessing might be affected when an edge } (u,x) \text{ changes}\}$

Dynamic Highway-Node Routing

change a few edge weights, server scenario



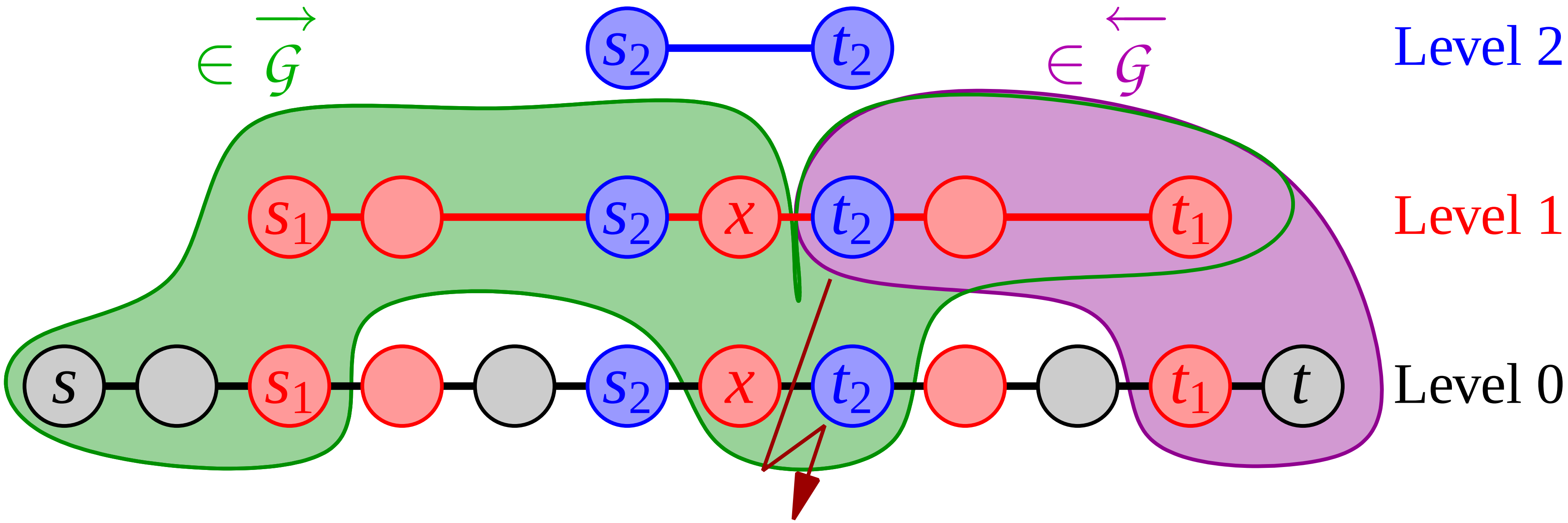
Dynamic Highway-Node Routing

change a few edge weights, mobile scenario

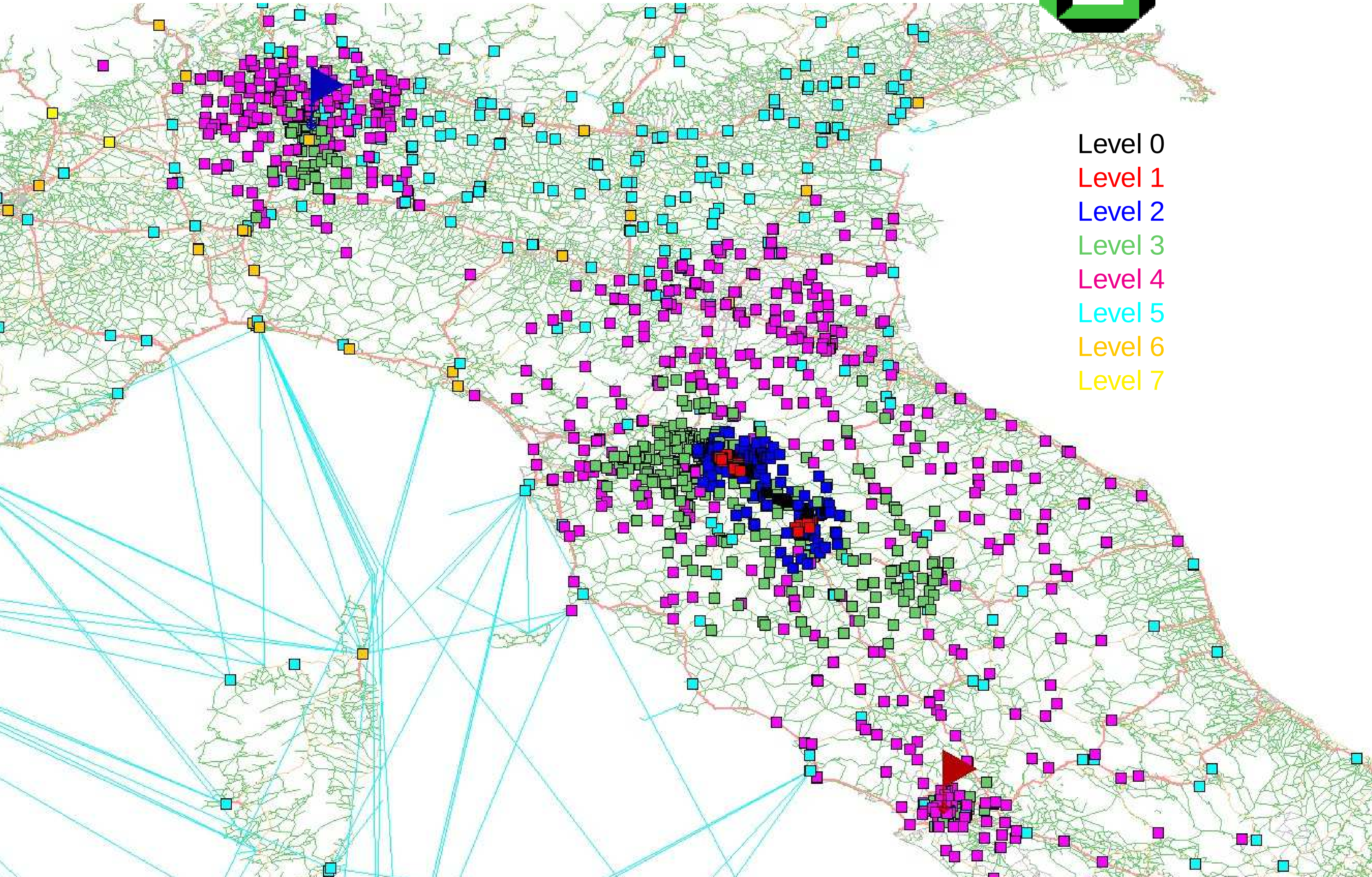
1. keep the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$
2. keep the overlay graphs
3. $C :=$ all changed edges
4. use the sets A_u^ℓ (considering edges in C) to determine for each node v a reliable level $r(v)$
5. during a query, at node v
   - do not use edges that have been created in some level $> r(v)$
   - instead, downgrade the search to level $r(v)$ (forward search only)

Dynamic Highway-Node Routing

change a few edge weights, mobile scenario



reliable levels: $r(x) = 0, \quad r(s_2) = r(t_2) = 1$


Level 0
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7

Dynamic Highway-Node Routing

change a few edge weights, mobile scenario

iterative variant (provided that only edge weight increases allowed)

1. keep everything (as before)
2. $C := \emptyset$
3. use the sets A_u^ℓ (considering edges in C) to determine for each node v a reliable level $r(v)$ (as before)
4. 'prudent' query (as before)
5. if shortest path P does not contain a changed edge, we are done
6. otherwise: add changed edges on P to C, **repeat** from 3.

Dynamic Highway-Node Routing

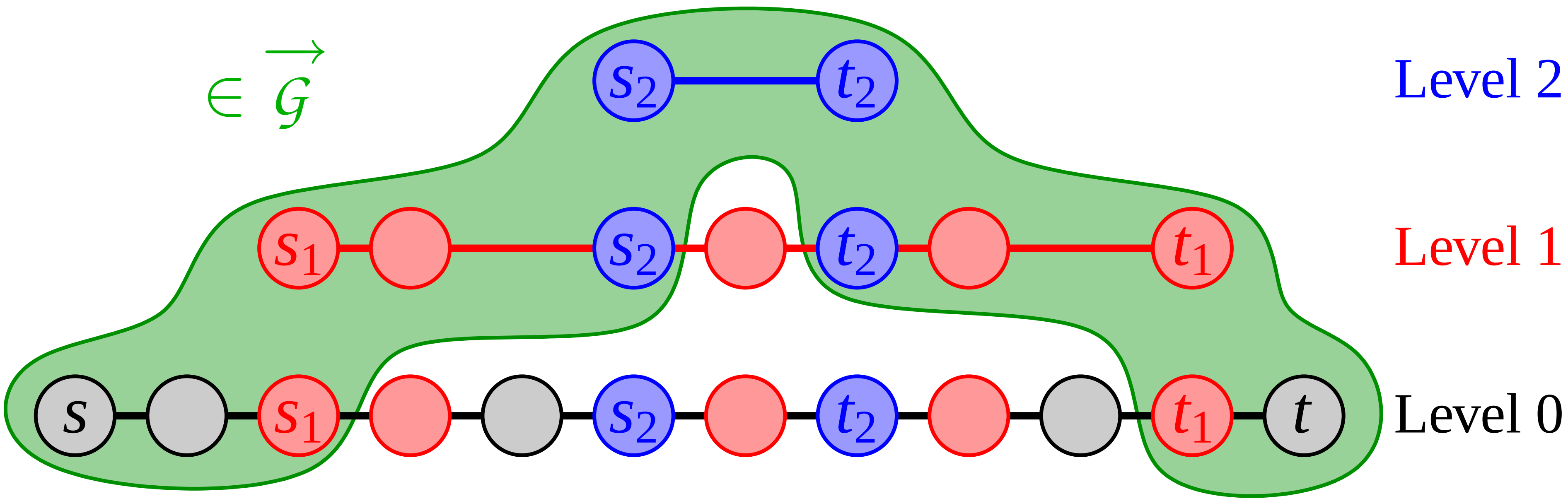
change a few edge weights, mobile scenario

		single pass	iterative		
|change set| (motorway edges)	affected queries	query time [ms]	query time [ms]	#iterations avg	#iterations max
1	0.4 %	2.3	1.5	1.0	2
10	5.8 %	8.5	1.7	1.1	3
100	40.0 %	47.1	3.6	1.4	5
1 000	83.7 %	246.3	25.3	2.7	9

Unidirectional Queries

1. keep everything (as before)
2. $C := \{$ some edge (t, x) $\}$
3. use the sets A_u^ℓ (considering edges in C) to determine for each node v a reliable level $r(v)$ (as before)
4. 'prudent' query (as before)

Unidirectional Queries



reliable levels: $r(t_1) = 0, \quad r(t_2) = 1$

Summary

- **efficient static approach**
  - fast preprocessing / fast queries — 15 min / 0.9 ms
  - outstandingly low memory requirements — 0.7 bytes/node $\rightsquigarrow$ 1.4 ms
- **can handle practically relevant dynamic scenarios**
  - change entire cost function — typically < 2 minutes
  - change a few edge weights
    - update data structures — 2 – 40 ms per changed edge

    OR
    - iteratively bypass traffic jams — e.g., 3.6 ms in case of 100 traffic jams

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numbers refer to the Western European road network with 18 million nodes and

to our 2.0 GHz AMD Opteron machine

Work in Progress

- find simpler / better ways to determine the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$

- parallelise the preprocessing

- implementation for a mobile device

Future Work

- handle a massive amount of updates

- deal with time-dependent scenarios

  (where edge weights depend on the time of day)

- allow multi-criteria optimisations