Transit Node Routing

based on Highway Hierarchies

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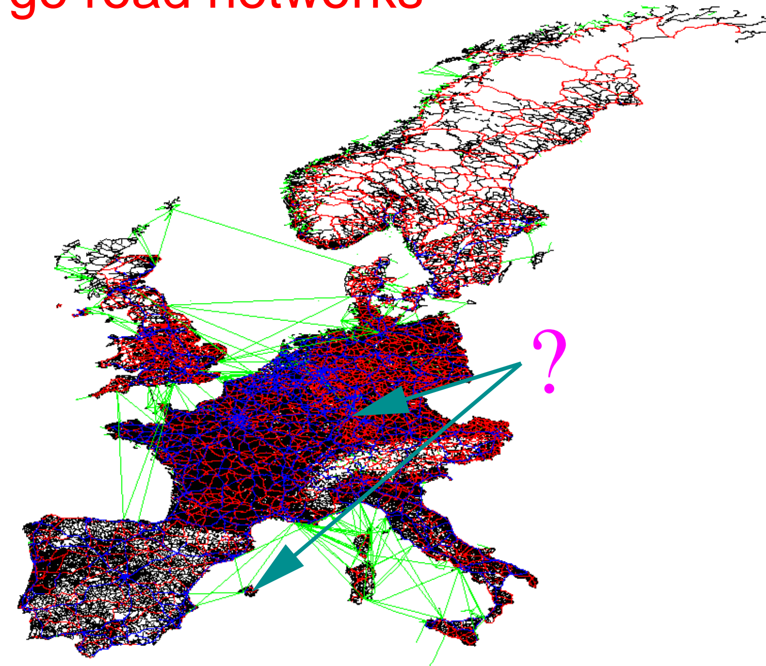
Route Planning

Goals:

- exact shortest (i.e. fastest) paths in large road networks
- fast queries
- fast preprocessing
- low space consumption



Applications:

- route planning systems in the internet
- car navigation systems
- ...

Motivation

‘Problem’:

existing solutions are already ‘too fast’.

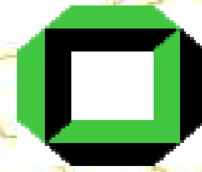
Example: perform a query (using hwy. hierarchies): $\approx$ 1 ms

visualise the path (using our Java application): $\approx$ 400 ms

Counter-Argument:

applications that require a lot of queries (and only a few paths)

- massive traffic simulations
- optimisations in logistics systems



Example:

Karlsruhe → Copenhagen

Example:

Karlsruhe $\longrightarrow$ Berlin

Example:

Karlsruhe $\longrightarrow$ Vienna

Example:

Karlsruhe → Munich

Example:

Karlsruhe → Rome

Example:

Karlsruhe → Paris

Example:

Karlsruhe $\longrightarrow$ London

Example:

Karlsruhe $\longrightarrow$ Brussels

Example:

Karlsruhe → Copenhagen

Example:

Karlsruhe $\longrightarrow$ Berlin

Example:

Karlsruhe → Vienna

Example:

Karlsruhe $\longrightarrow$ Munich

Example:

Karlsruhe $\longrightarrow$ Rome

Example:

Karlsruhe $\longrightarrow$ Paris

Example:

Karlsruhe $\longrightarrow$ London

Example:

Karlsruhe → Brussels

First Observation

For long-distance travel: leave current location

via one of only a few ‘important’ traffic junctions,

called **access points**

($\leadsto$ we can afford to store all access points for each node)

[in Europe: about 10 access points per node on average]

Example:

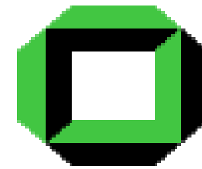
Karlsruhe → Berlin

Example:

Karlsruhe $\longrightarrow$ Berlin

Example:

Karlsruhe $\longrightarrow$ Berlin



Second Observation

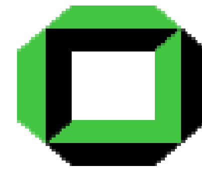
Each access point is relevant for several nodes. $\leadsto$

union of the access points of all nodes is small,

called **transit node set**

($\leadsto$ we can afford to store the distances between all transit node pairs)

[in Europe: about 10 000 transit nodes]



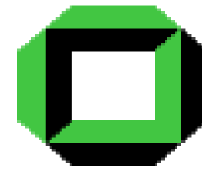
Transit Node Routing

Preprocessing:

- identify transit node set $\mathcal{T} \subseteq V$
- compute complete $|\mathcal{T}| \times |\mathcal{T}|$ distance table
- for each node: identify its access points (mapping $A : V \to 2^{\mathcal{T}}$),
  store the distances

Query (source s and target t given): compute

$$d_{\text{top}}(s,t) := \min\{d(s,u) + d(u,v) + d(v,t) : u \in A(s), v \in A(t)\}$$



Transit Node Routing

Locality Filter:

local cases must be filtered ($\leadsto$ special treatment)

$L : V \times V \rightarrow \{\text{true}, \text{false}\}$

$\neg L(s,t)$ implies $d(s,t) = d_{\text{top}}(s,t)$

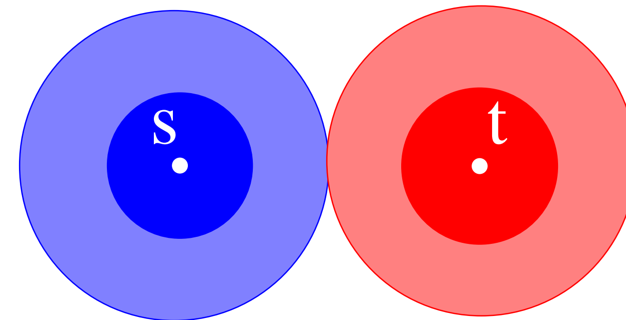
Example

Related Work

- **separator-based** implementation [Müller et al. 2006]
  - determine separator nodes (= transit nodes)
  - partition the graph into small components
  - access points of node u: border nodes of u's component
  - locality filter: "same component?"

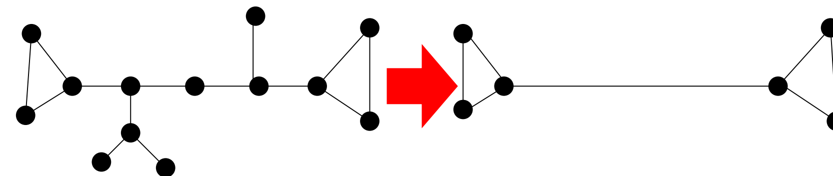
- **grid-based** implementation [Bast, Funke, Matijevic 2006]
  - compute geometric subdivision of the network into cells
  - access points: border nodes needed for 'long-distance' travel
  - transit nodes: union of all access points
  - locality filter: "less than a certain number of grid cells apart?"

Our Approach: Highway Hierarchies[1]

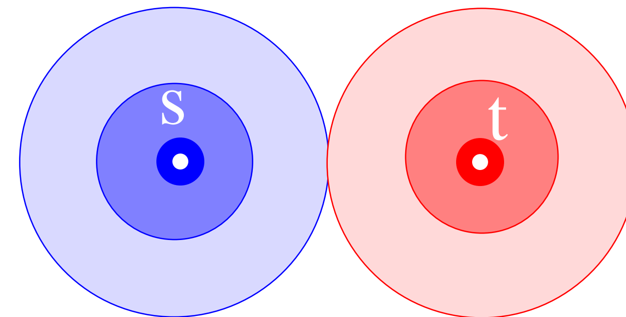


- complete search within a local area
- search in a (thinner) highway network

  = minimal graph that preserves all shortest paths



- contract network, e.g.,
- iterate ⇝ highway hierarchy



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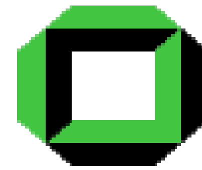
[1] presented at ESA 2005 and ESA 2006



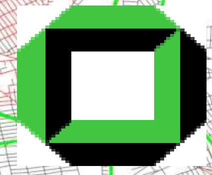
Example: Karlsruhe

Level 0

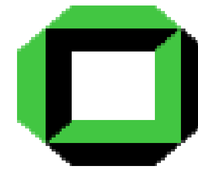
Level 1

Level 2

Level 3



Example: New York City



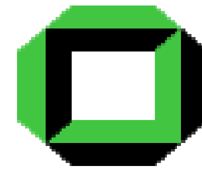
Local Area

- choose neighbourhood radius $r(s)$

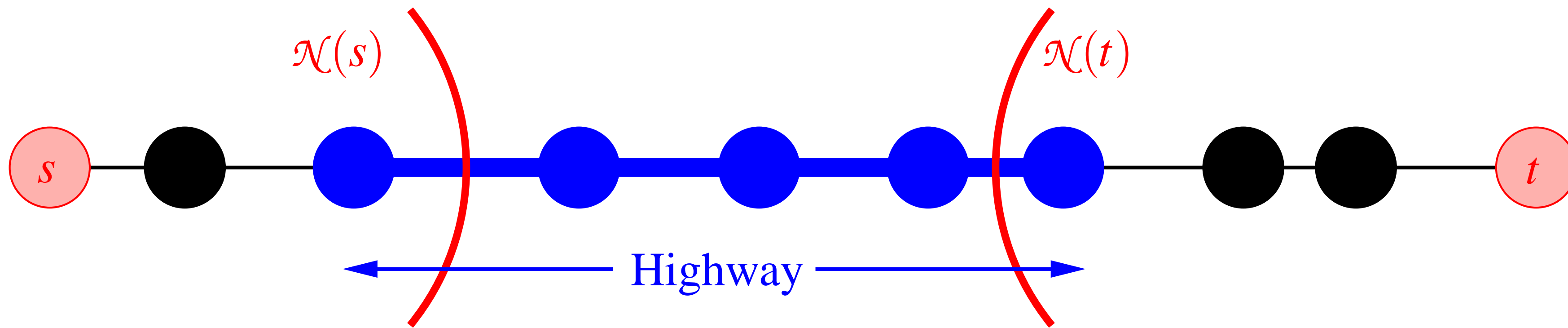
  (by a heuristic)

- define neighbourhood of s

$$\mathcal{N}(s) := \{v \in V \mid d(s,v) \leq r(s)\}$$



Highway Network

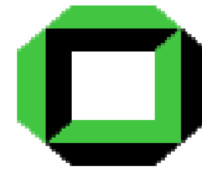


Edge (u, v) belongs to highway network *iff* there are nodes s and t s.t.

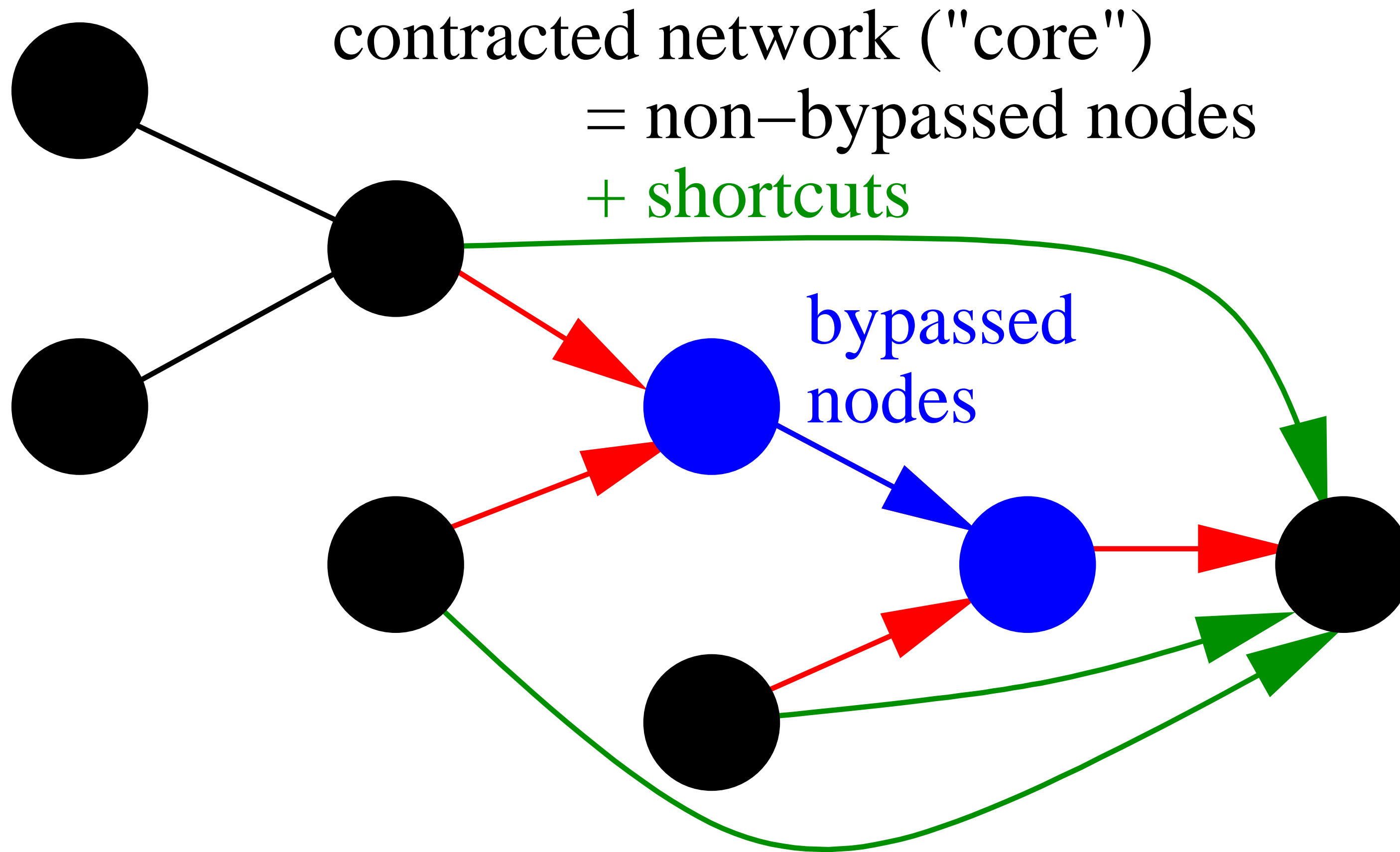
- (u, v) is on the "*canonical*" shortest path from s to t

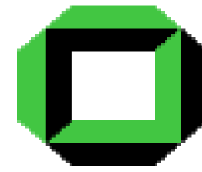
and

- (u, v) is not entirely within $\mathcal{N}(s)$ or $\mathcal{N}(t)$

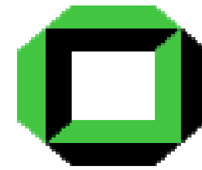


Contraction





Distance Table: Search Space Example

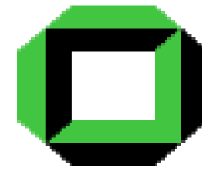


Distance Table

- Compute an all-pairs distance table for the core of the topmost level ℓ. 13 465 $\times$ 13 465 entries

- Abort the search when all entrance points in the core of level ℓ have been encountered. $\approx$ 55 for each direction

- Use the distance table to bridge the gap. $\approx$ 55 $\times$ 55 entries



HH-based Transit Node Routing

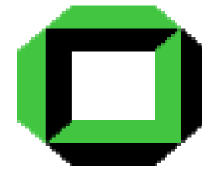
- Compute an all-pairs distance table for the core of the topmost level ℓ. — 13 465 × 13 465 entries

  core of the topmost level ℓ: transit node set $\mathcal{T}$

- Abort the search when all entrance points in the core of level ℓ have been encountered. — ≈ 55 for each direction

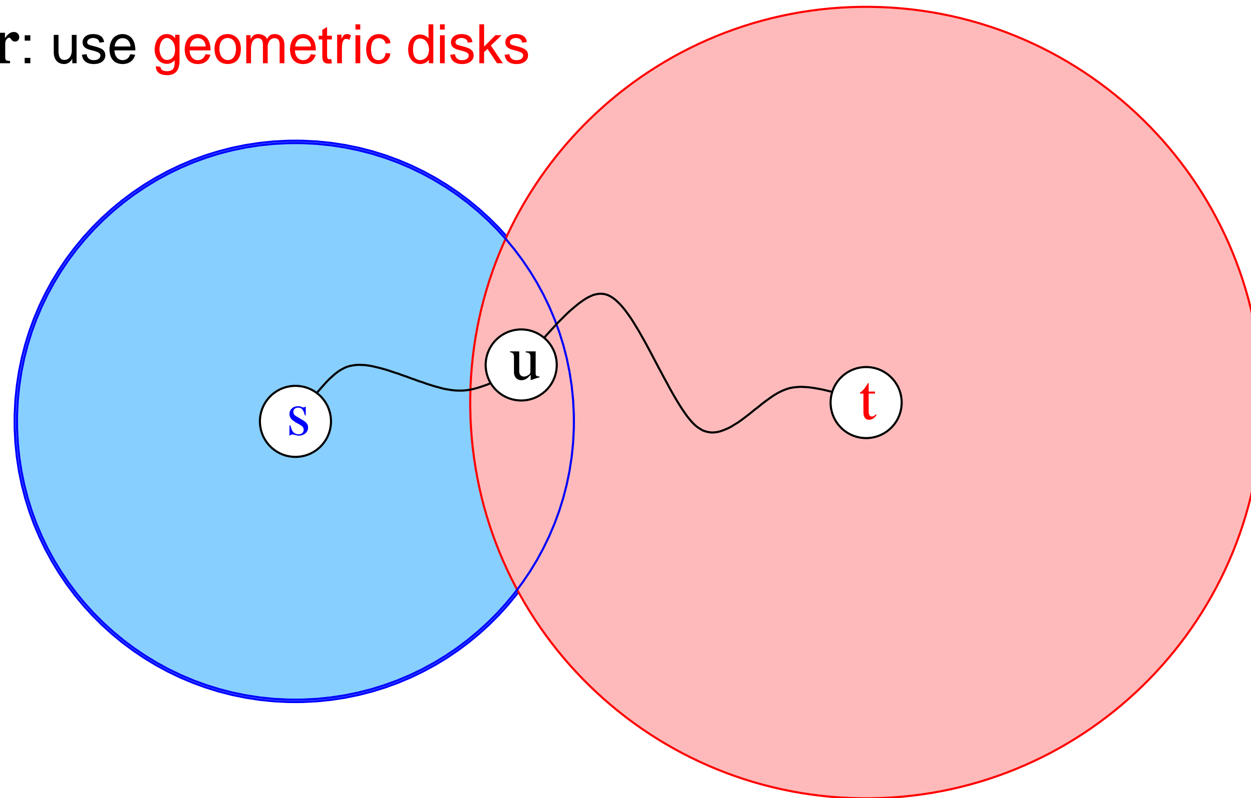
  do not ‘search’, just perform look-ups

- Use the distance table to bridge the gap. — ≈ 55 × 55 entries



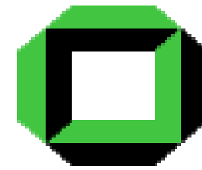
Missing Pieces

1. **locality filter**: use geometric disks



$$d(s,t) = d_u(s,t) < d_{top}(s,t)$$

$L(s,t) :=$ "disks of s and t overlap"

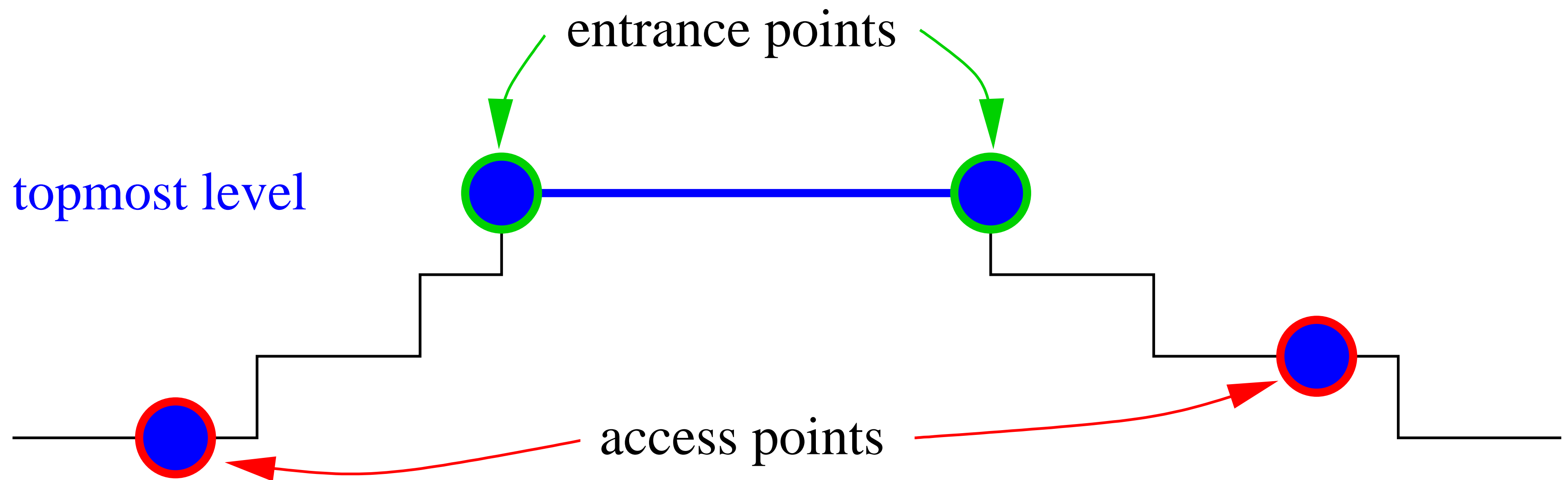


Missing Pieces

2. too many 'entrance points' (55)

solution: fall back on comparatively few 'access points' (10)

(motivated by the observations from [Bast, Funke, Matijevic 2006])



Missing Pieces

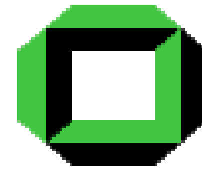
3. compute top distance table in the original graph

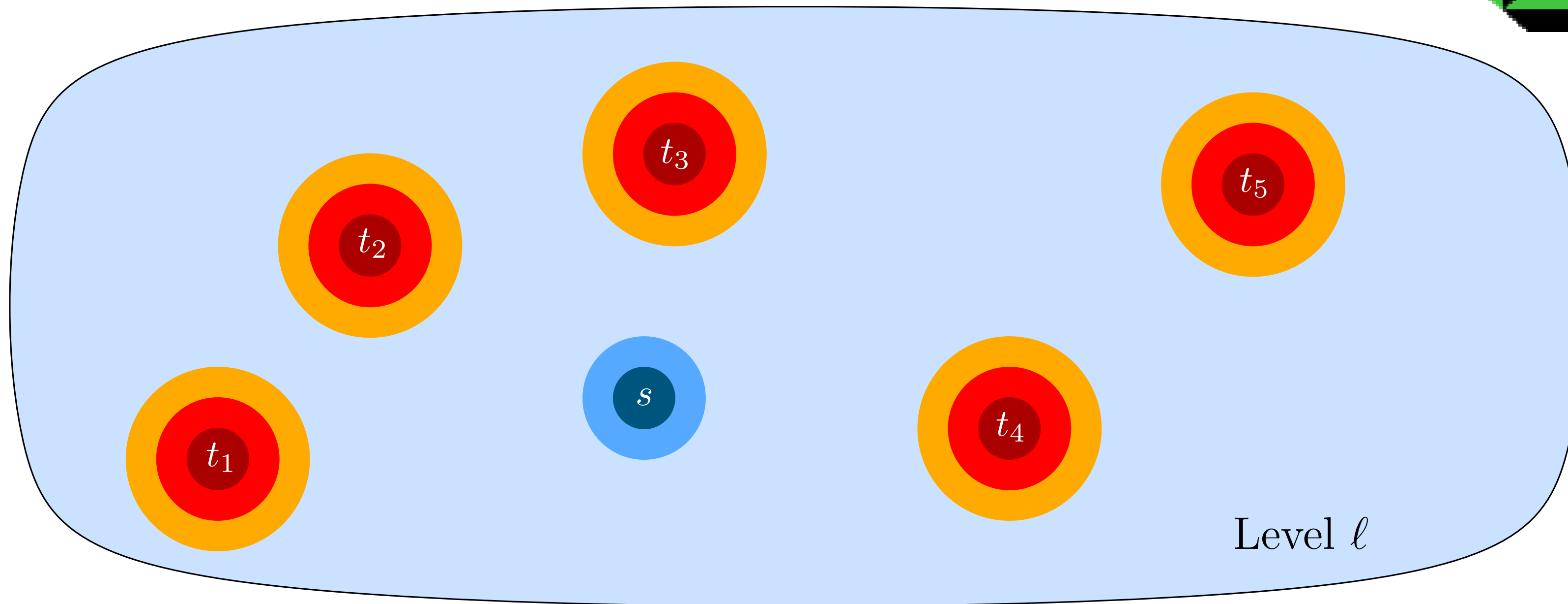
solution: [Knopp, S, S, Schulz, Wagner 2007]

"Computing Many-to-Many Shortest Paths

Using Highway Hierarchies"

(e.g. $10\,000 \times 10\,000$ table in one minute)





- for each $t \in \mathcal{T}$, perform backward search up to top level ℓ,
  store search space entries $(t, u, d(u, t))$
- arrange search spaces: group entries by u
- for each $s \in \mathcal{T}$, perform forward search,
  at each node u, scan all entries $(t, u, d(u, t))$ and
  compute $d(s, u) + d(u, t)$

Second Layer

(to deal with medium range queries)

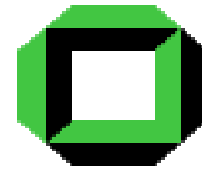
- secondary transit node set $\mathcal{T}_2 \supset \mathcal{T}$
- secondary access mapping $A_2 : V \rightarrow 2^{\mathcal{T}_2}$
- secondary dist. table $\{d(u,v) : u,v \in \mathcal{T}_2 \wedge d(u,v) \neq d_{\text{top}}(u,v)\}$
- secondary locality filter L_2

$\neg L_2(s,t)$ **implies**

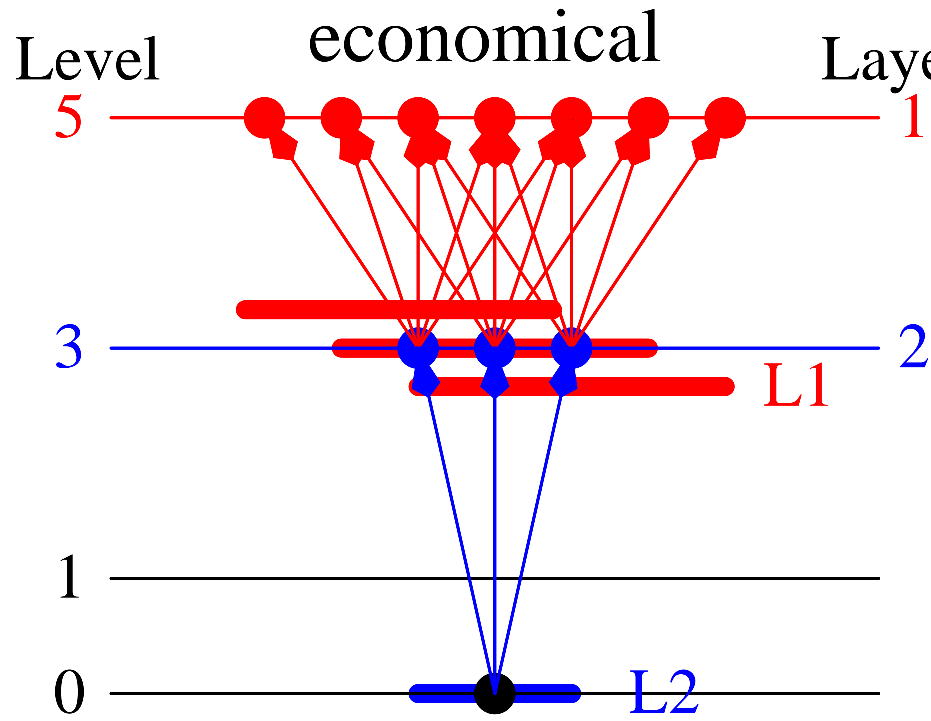
$d(s,t) = d_{\text{top}}(s,t)$

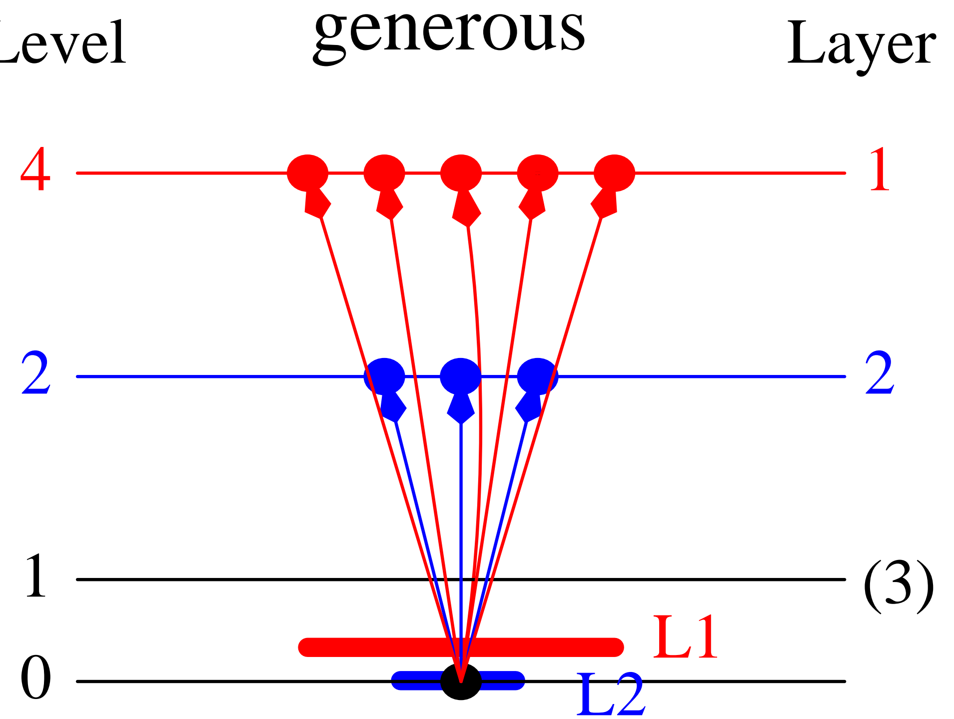
OR

$d(s,t) = \min\{d(s,u)+d(u,v)+d(v,t) : u \in A_2(s), v \in A_2(t)\}$



Two Concrete Variants





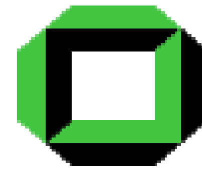
Preprocessing

- for each secondary transit node $t \in \mathcal{T}_2$:
  - perform backward highway search
  - stop at primary transit nodes ($\rightsquigarrow$ set of backward access points)
  - store search space entries $(t, u, d(u, t))$
- arrange search spaces

Preprocessing

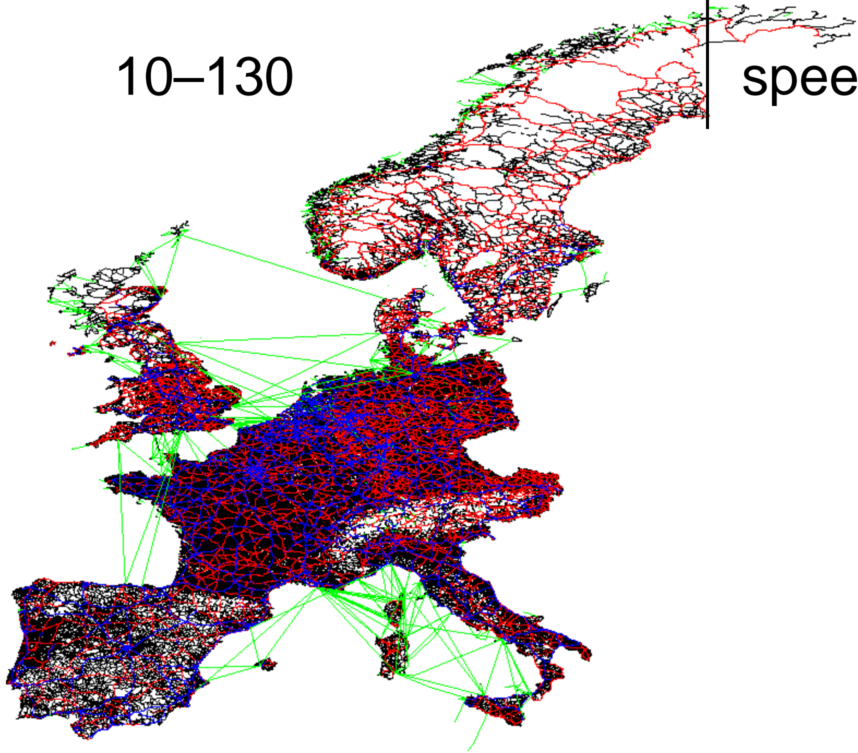
- for each secondary transit node $s \in \mathcal{T}_2$:
  - perform forward highway search
  - stop at primary transit nodes ($\leadsto$ set of forward access points)
  - at each node u and for each search space entry $(t, u, d(u,t))$:
    - ∗ compute distance $d_u(s,t) := d(s,u) + d(u,t)$ via u
    - ∗ compute distance $d_{\text{top}}(s,t)$ via top layer
    - ∗ if $d_u(s,t) < d_{\text{top}}(s,t)$ then
      - · add entry $d_u(s,t)$ to the secondary distance table
      - · ensure that the disks around s and t contain u

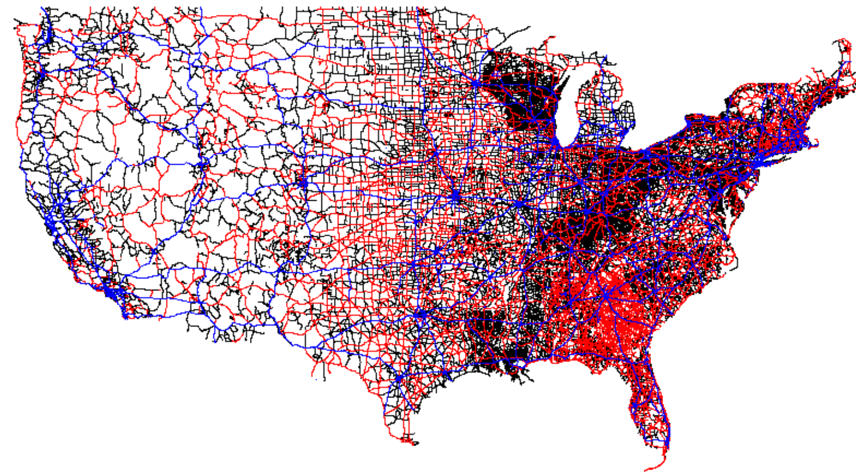
(use similar procedure for lower layers)

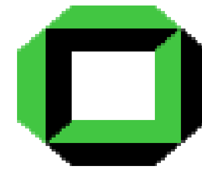


Experiments

W. Europe (PTV)	Our Inputs	USA (TIGER/Line)
18 010 173	#nodes	23 947 347
42 560 279	#directed edges	58 333 344
13	#road categories	4
10–130	speed range [km/h]	40–100





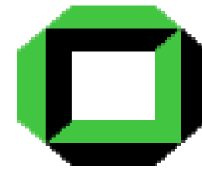


Preprocessing

metric	variant	layer 1 $\lvert\mathcal{T}\rvert$	$\lvert A\rvert$	layer 2 $\lvert\mathcal{T}_2\rvert$	$\lvert A_2\rvert$	space [B/node]	time [h]
USA time	eco	12 111	6.1	184 379	4.9	111	0:59
USA time	gen	10 674	5.7	485 410	4.2	244	3:25
USA dist	eco	15 399	17.0	102 352	10.9	171	8:58
EUR time	eco	8 964	10.1	118 356	5.5	110	0:46
EUR time	gen	11 293	9.9	323 356	4.1	251	2:44
EUR dist	eco	11 610	20.3	69 775	13.1	193	7:05

Query

	metric	variant	layer 1 [%] wrong	layer 1 [%] cont'd	layer 2 [%] wrong	layer 2 [%] cont'd	time
USA	time	eco	0.14	1.13	0.0064	0.2780	11.5 μs
		gen	0.11	0.80	0.0014	0.0138	4.9 μs
	dist	eco	1.57	8.10	0.0489	2.2352	87.5 μs
EUR	time	eco	0.54	2.87	0.0092	0.5843	13.4 μs
		gen	0.26	1.35	0.0015	0.0190	5.6 μs
	dist	eco	4.68	18.32	0.1761	4.2764	107.4 μs



Local Queries: USA, travel time metric

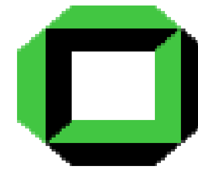
Query Time [µs]

1000 300 100 40 20 10 5

economical
generous

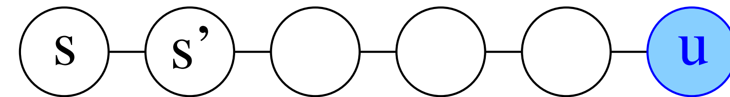
2^5 2^6 2^7 2^8 2^9 2^{10} 2^{11} 2^{12} 2^{13} 2^{14} 2^{15} 2^{16} 2^{17} 2^{18} 2^{19} 2^{20} 2^{21} 2^{22} 2^{23} 2^{24}

Dijkstra Rank

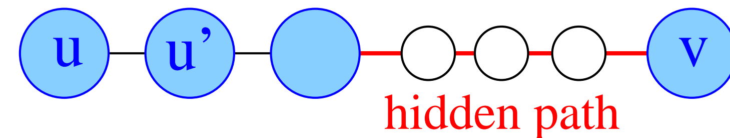


Determine Shortest Paths

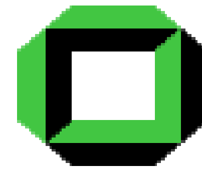
- from source s, look iteratively for the next adjacent node s' that leads to fwd acc pnt u
  $d(s,s')+d(s',u)=d(s,u)$



- analogously, from target to bwd acc pnt
- from fwd acc pnt u, look iteratively for the next adjacent node u' in the topmost level that leads to bwd acc pnt v
  $d(u,u')+d(u',v)=d(u,v)$
  (if necessary, use a hidden path instead)

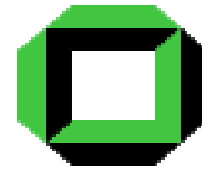


- unpack shortcuts
- if path not through top layer, fall back on highway query



Determine Shortest Paths

	preproc. [min]	space [MB]	query [μs]	# hops (avg.)
USA	4:04	193	258	4 537
EUR	7:43	188	155	1 373



Summary

- **insight**: human intuition is applicable in this case
- extremely fast queries (down to 5 μs)
- handles all kinds of queries (local/global) (5–20 μs)
- moderate preprocessing times (down to 46 min)
- output shortest paths quickly (down to 155 μs)

Negative Example

Future Work

- **select better transit node set**

  (e.g. some combination with the separator-based approach)

- more flexible implementation (to better deal with the distance metric)

- more space-efficient implementation

  (e.g. store access points only at the core-1 nodes)

- use access points as landmarks to guide local search

- improve locality filter (get rid of geometry?)