Accurate High-Performance Route Planning

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`http://algo2.iti.uka.de/schultes/hwy/`

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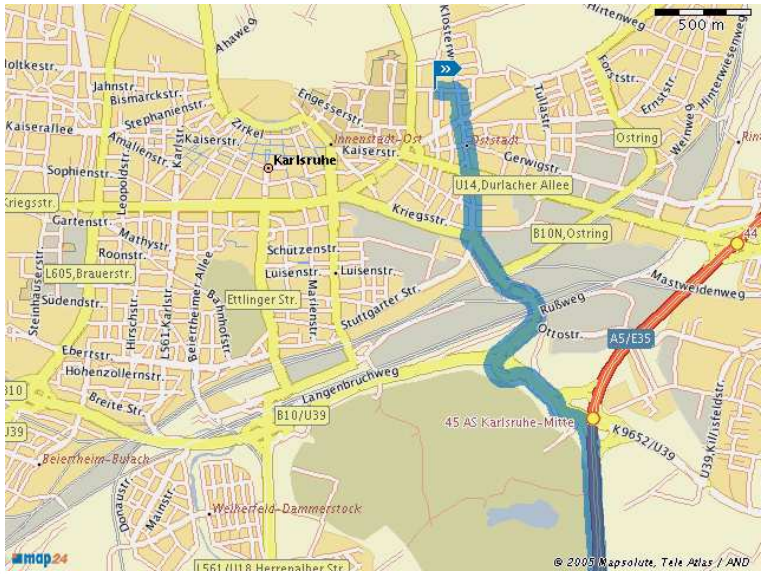
How do I get there from here ?

Applications

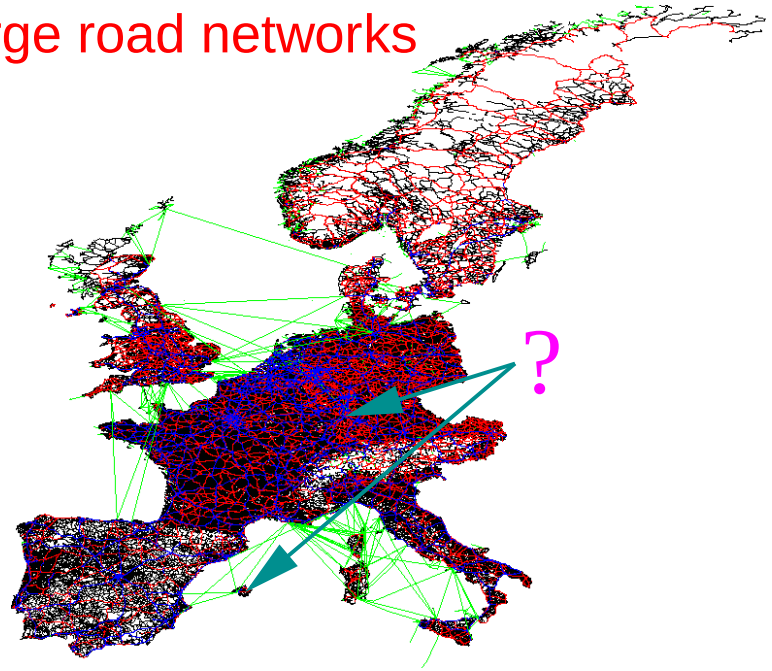
- route planning systems
  in the internet
  (e.g. `www.map24.de`)
- car navigation systems
- . . .



Goals

- exact shortest (i.e. fastest) paths in large road networks
- fast queries
- fast preprocessing
- low space consumption
- scale-invariant,

  i.e., optimised not only for long paths

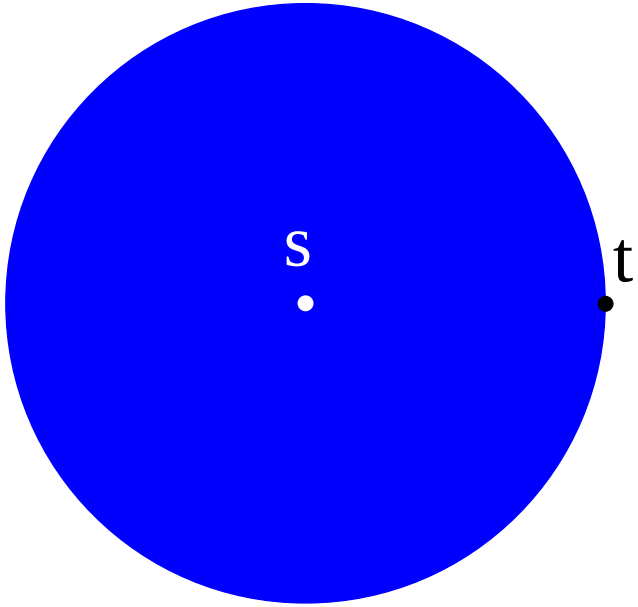


Related Work

method	query	prepr.	space	scale	source
basic A^*	$-$	$++$	$++$	$+$	[Hart et al. 68]
bidirected	$-$	$++$	$++$	$+$	[Pohl 71]
heuristic hwy hier.	$+$	$++$	$+$	$+$	[commercial]
separator hierarchies	o	?	$-$	$-$	[several groups 02]
geometric containers	$++$	$--$	$+$	$+$	[Wagner et al. 03]
bitvectors	$++$	$-$	o	$-$	[Lauther. . . 04]
landmarks	$+$	$++$	$-$	$-$	[Goldberg et al. 04]
landmarks $+$ reaches	$++$	o	o	o	[Goldberg et al. 06]
highway hierarchies	$++$	$+$	$+$	$+$	here

DIJKSTRA's Algorithm

Dijkstra



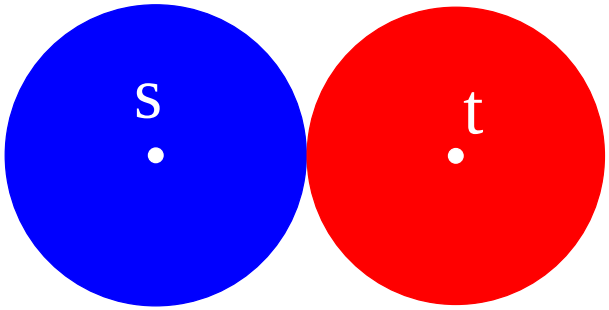
not practicable

for large road networks

(e.g. Western Europe:

≈ 18 000 000 nodes)

bidirectional Dijkstra



improves the running time,

but still too slow

Naive Route Planning

1. Look for the next reasonable motorway

Naive Route Planning

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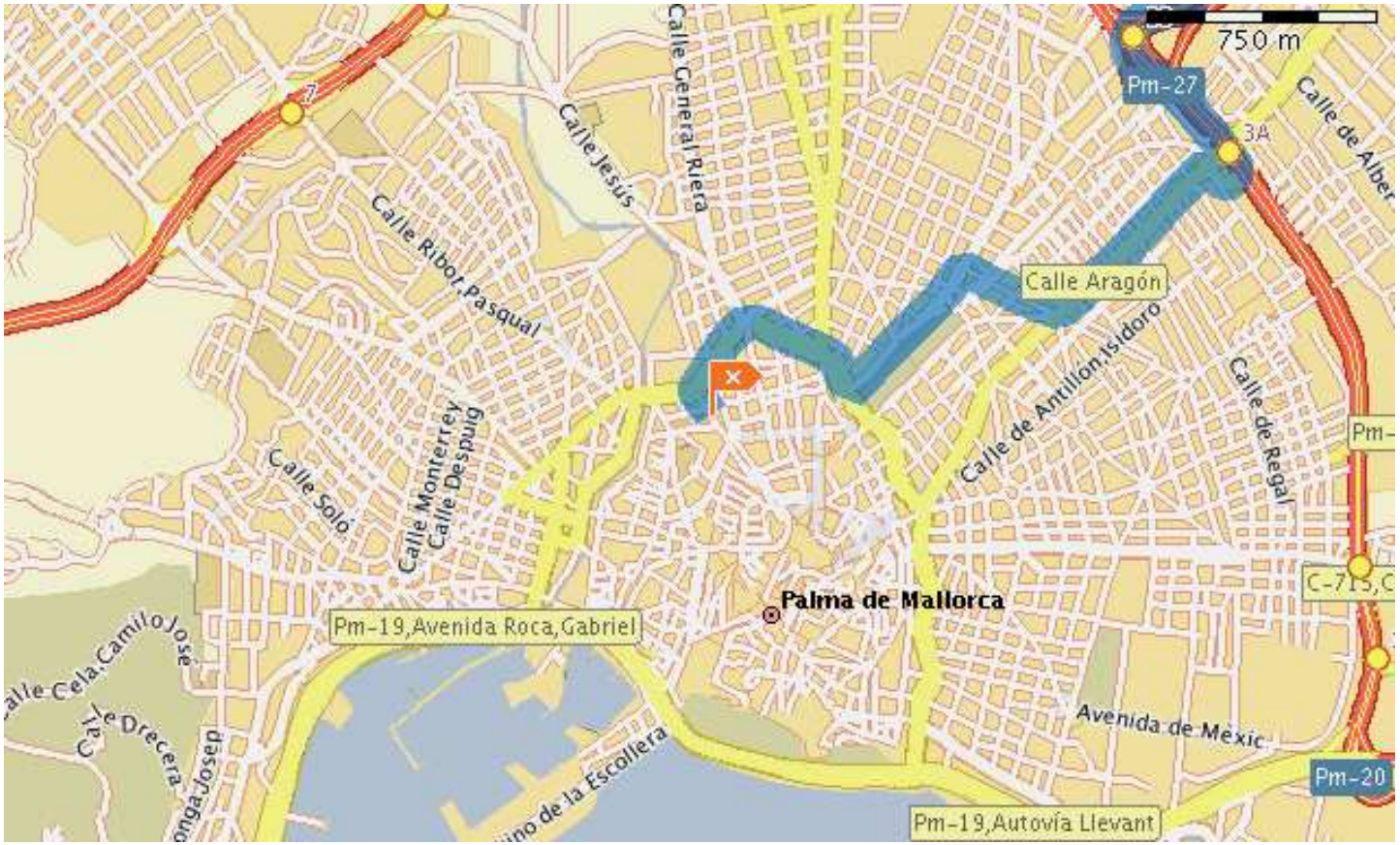
2. Drive on motorways to a location close to the target

Naive Route Planning

1. Look for the next reasonable motorway
2. Drive on motorways to a location close to the target
3. Search the target starting from the motorway exit



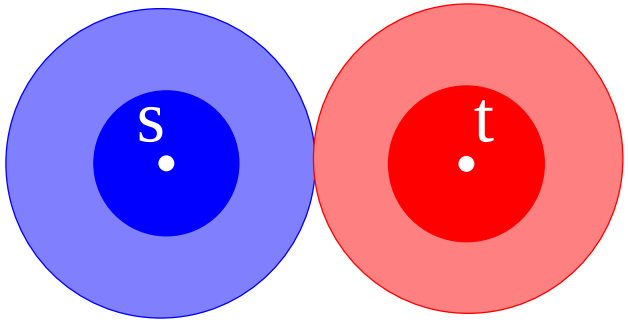
Commercial Systems

1. Search from the source and target node (‘bidirectional’)
   within a certain radius (e.g. 20 km),
   consider all roads
2. Continue the search within a larger radius (e.g. 100 km),
   consider only national roads and motorways
3. Continue the search,
   consider only motorways

fast, but not exact

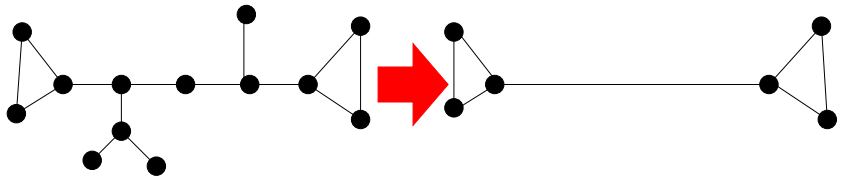
Exact Highway Hierarchies

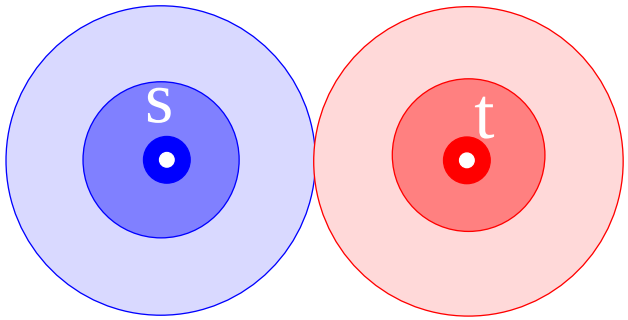
- complete search within a local area
- search in a (thinner) highway network



= minimal graph that preserves all shortest paths

- contract network, e.g.,
- iterate $\rightsquigarrow$ highway hierarchy





A Meaning of "Local"

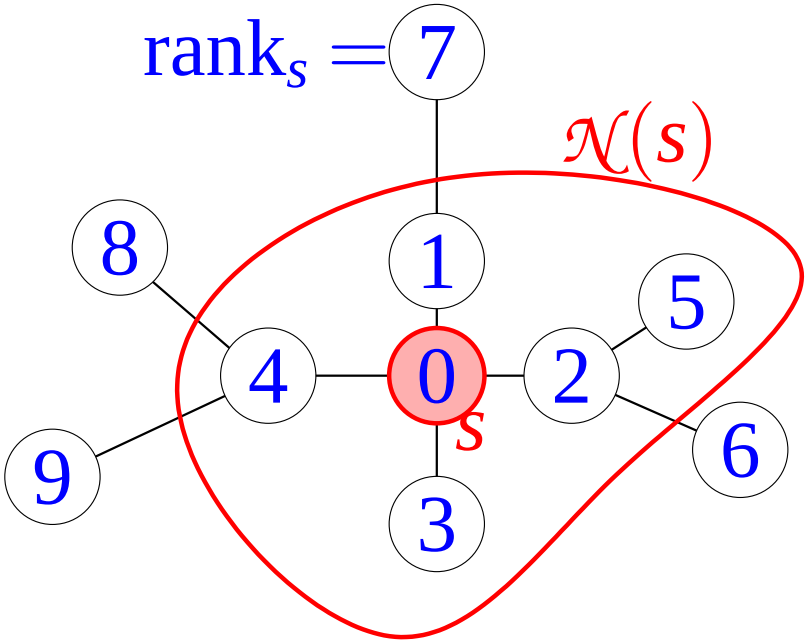
- choose neighbourhood radius $r(s)$

  e.g. distance to the H-closest node for a fixed parameter H

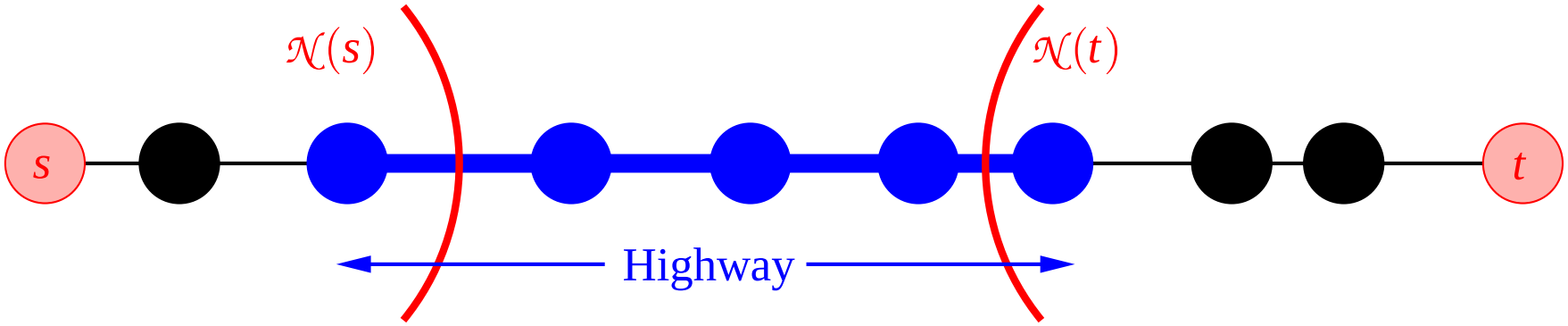
- define neighbourhood of s:

  $\mathcal{N}(s) := \{v \in V \mid d(s,v) \leq r(s)\}$

- example for $H = 5$



Highway Network



Edge (u,v) belongs to highway network *iff* there are nodes s and t s.t.

- (u,v) is on the "*canonical*" shortest path from s to t

and

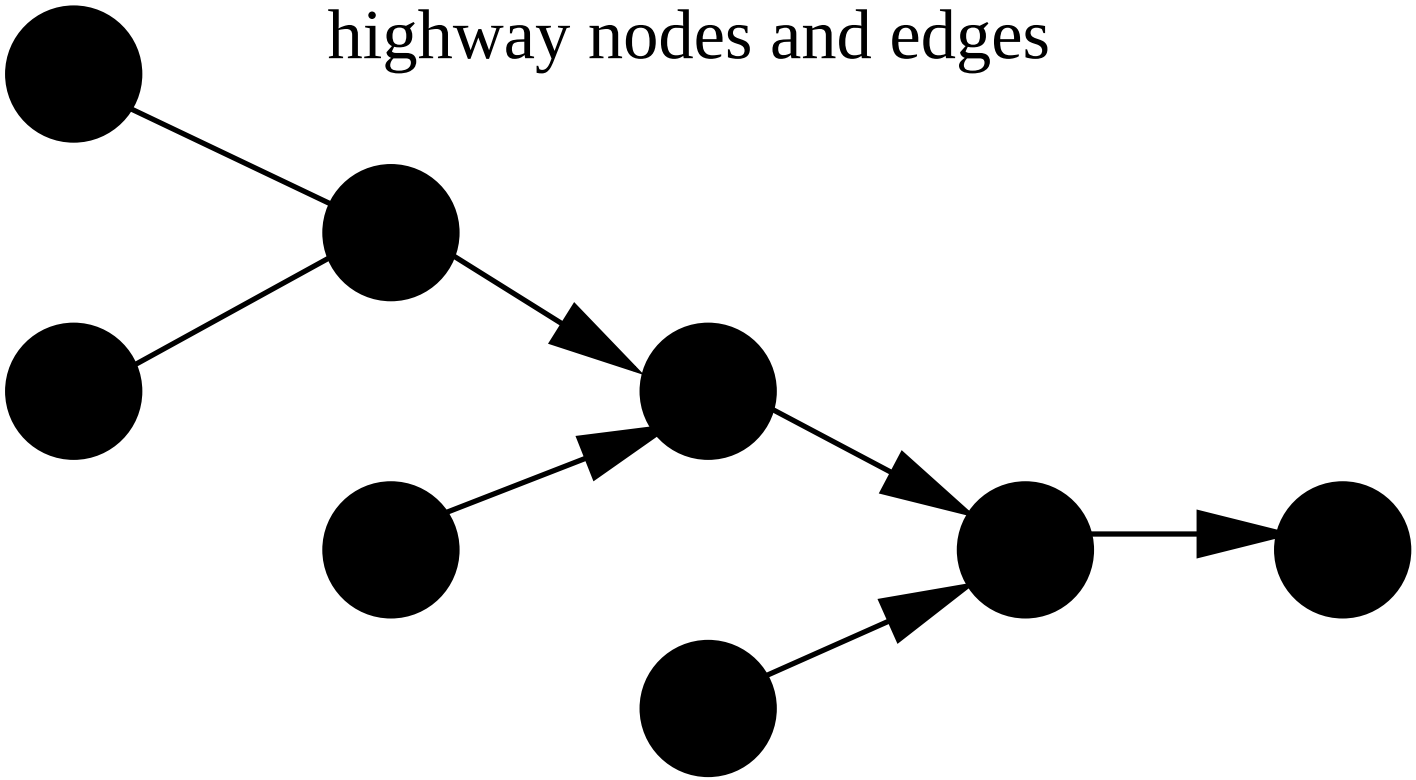
- $v \notin \mathcal{N}(s)$

and

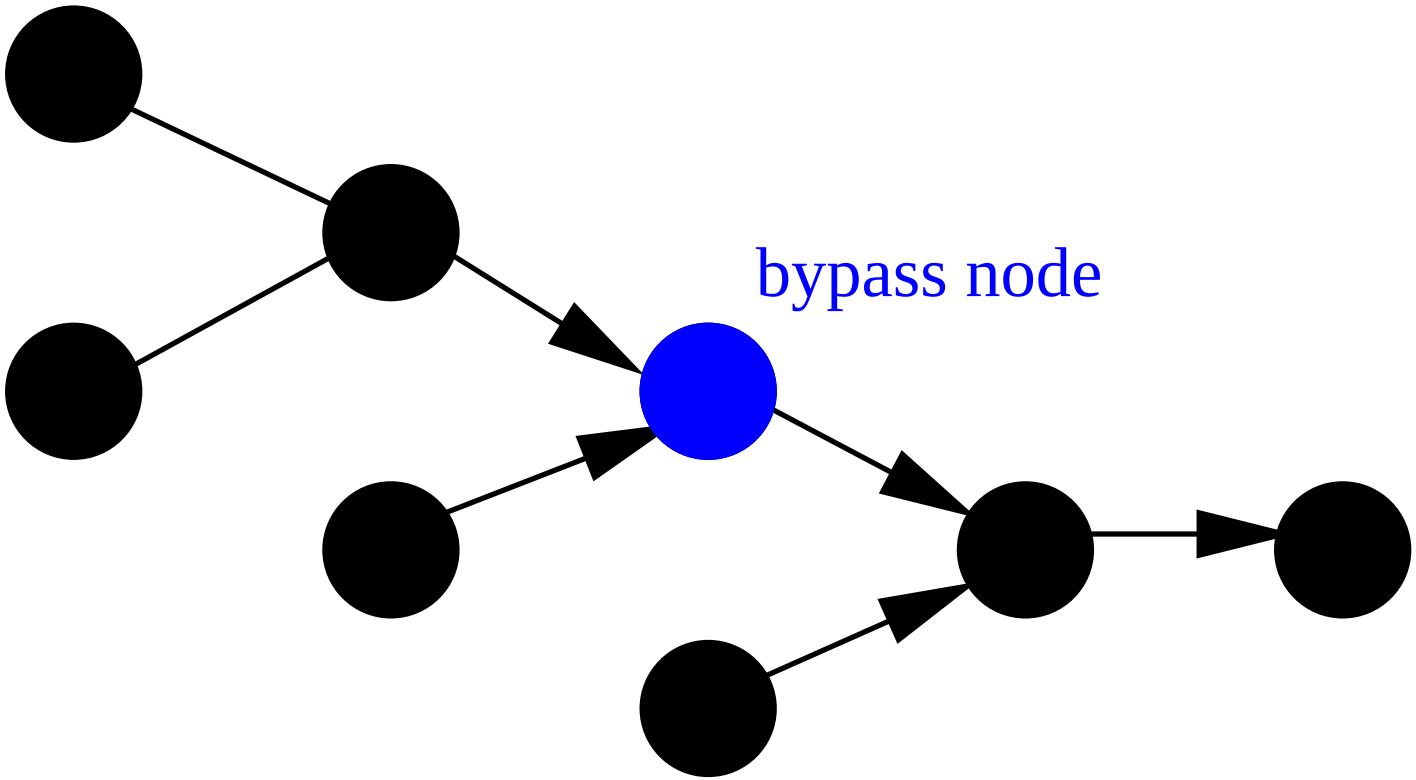
- $u \notin \mathcal{N}(t)$

Contraction

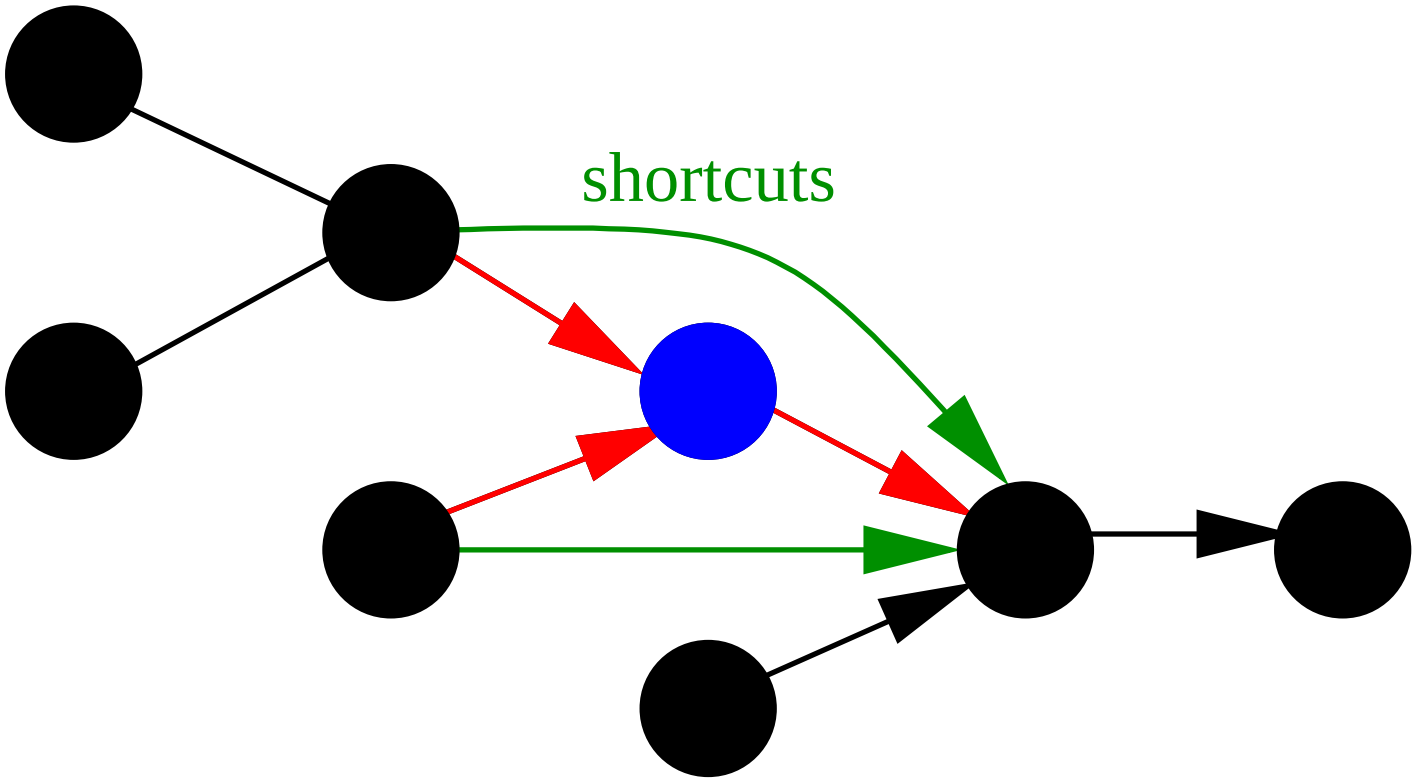
highway nodes and edges



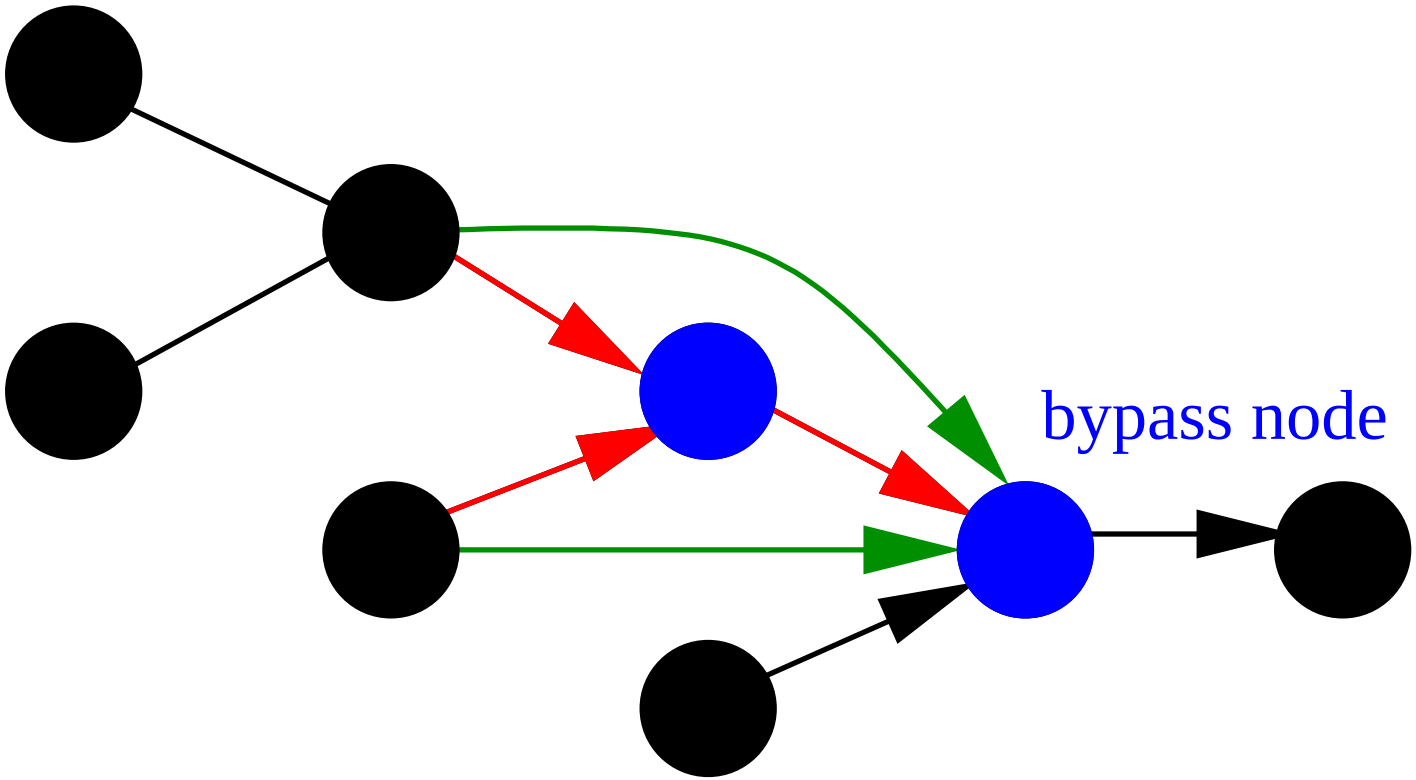
Contraction



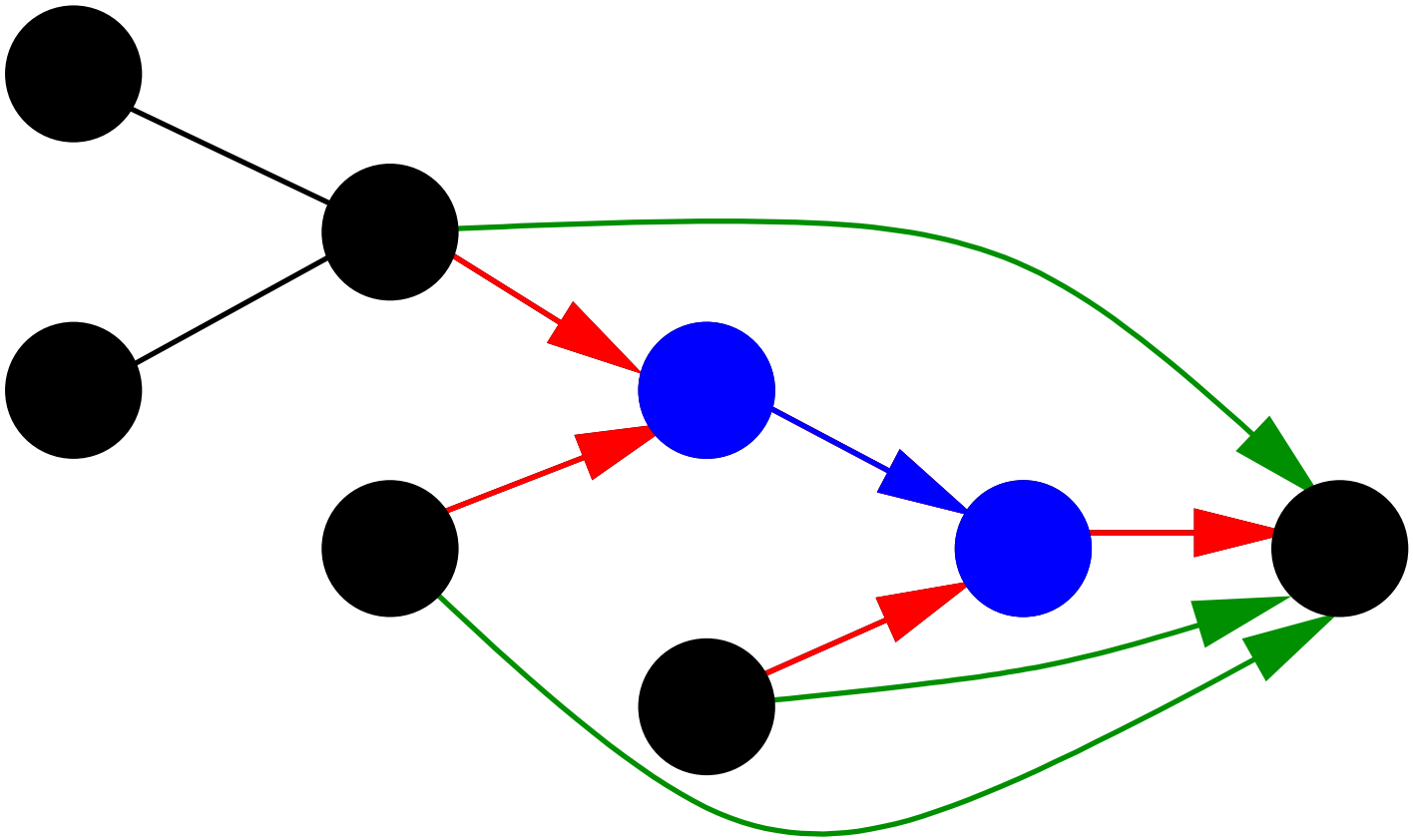
Contraction



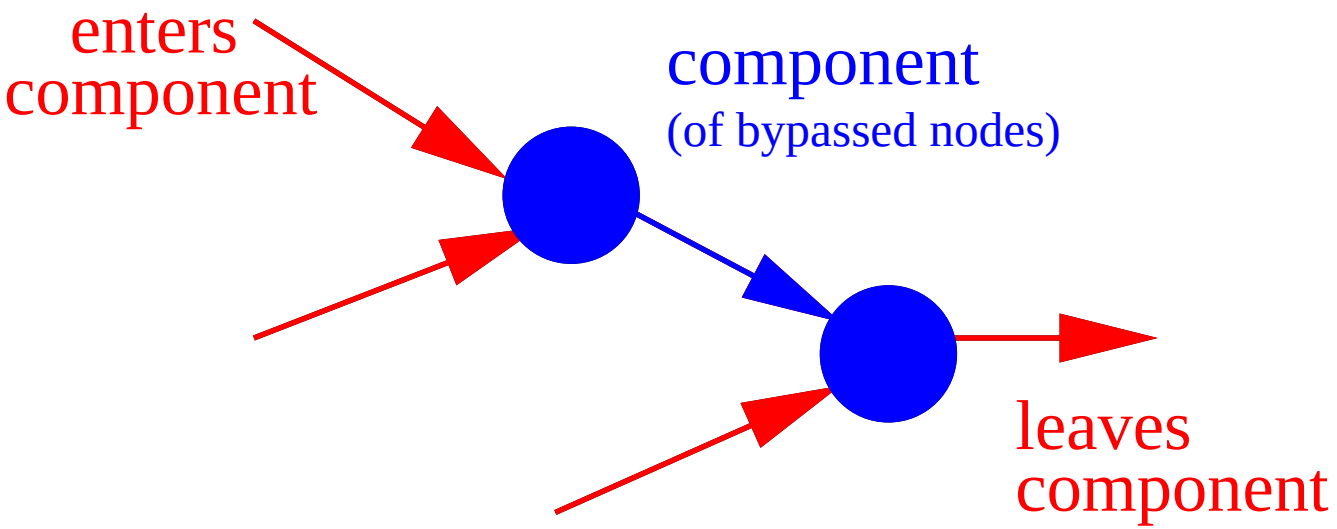
Contraction



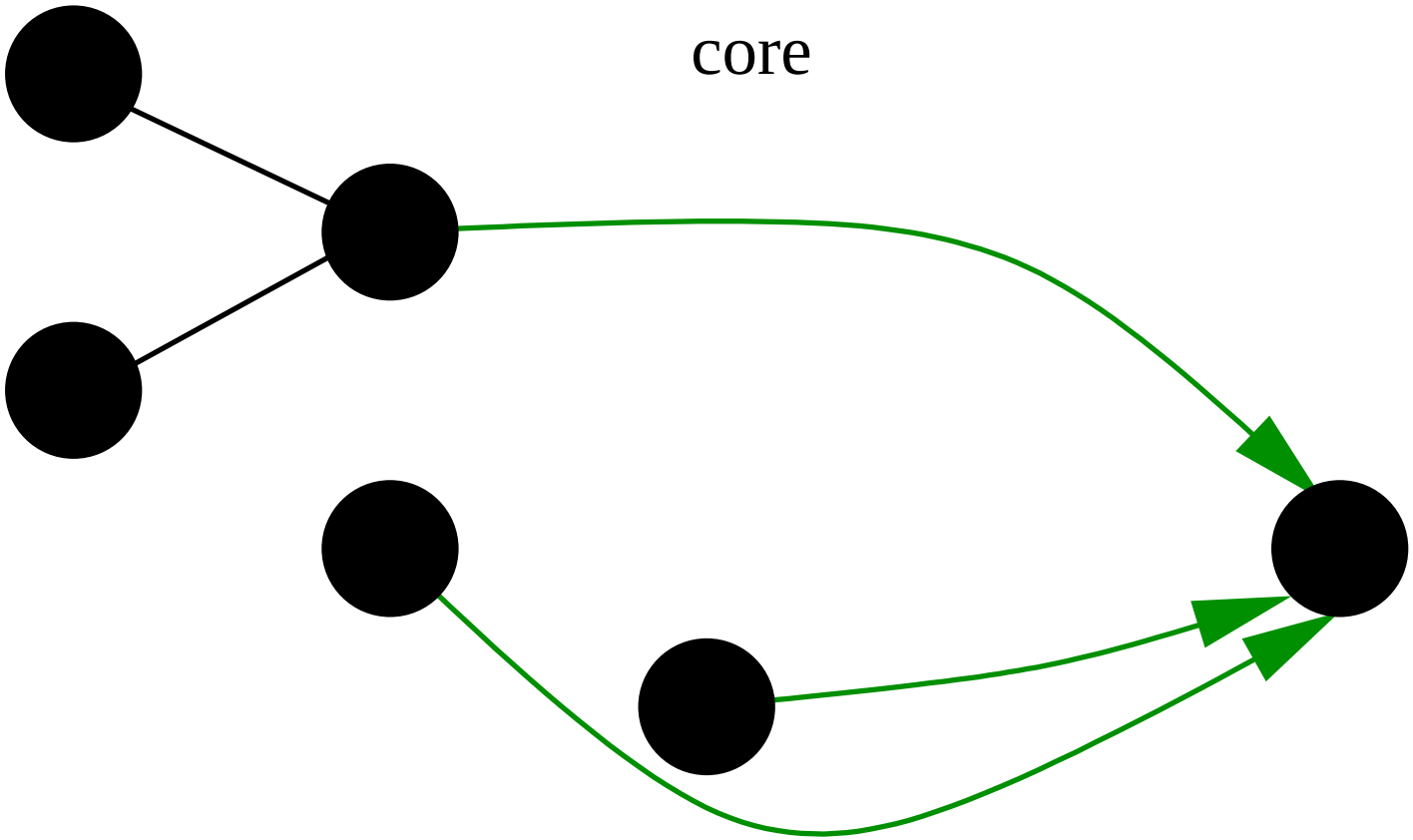
Contraction



Contraction



Contraction



Contraction

Which nodes should be bypassed?

Use some heuristic taking into account

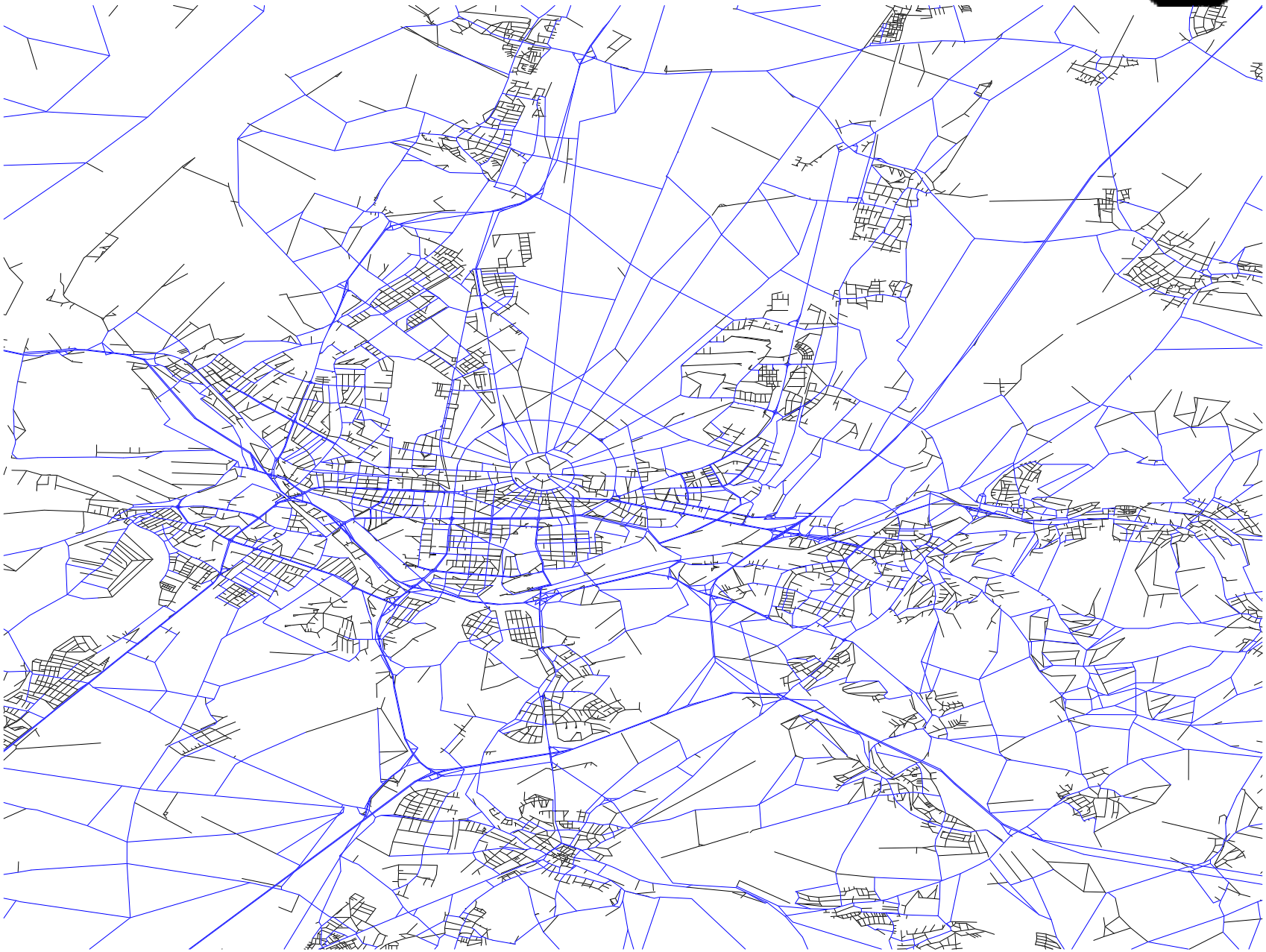
- the number of shortcuts that would be created and
- the degree of the node.

Construction

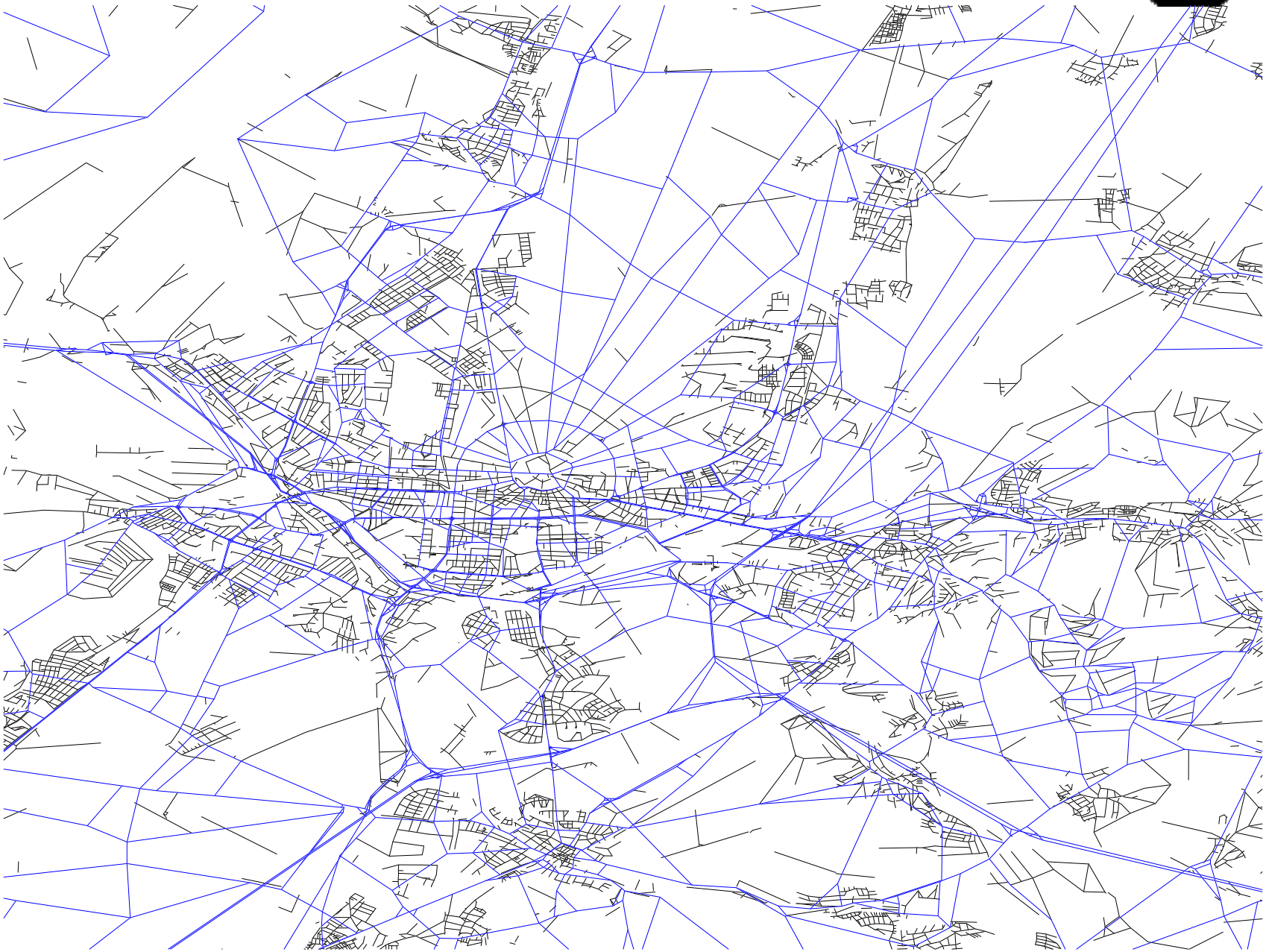
Example: Western Europe, bounding box around Karlsruhe

Complete Road Network

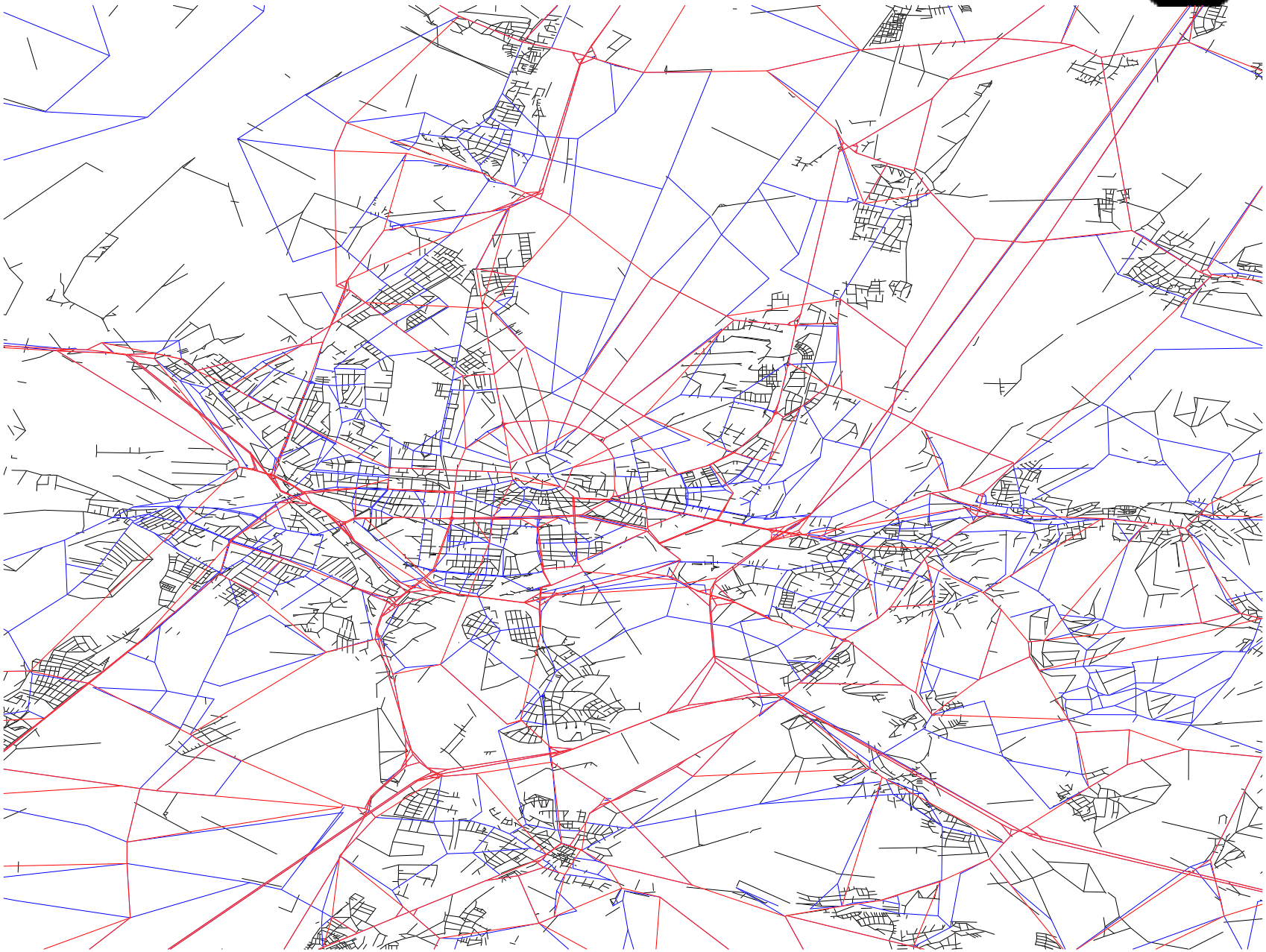
Highway Network



Contracted Network (Core)



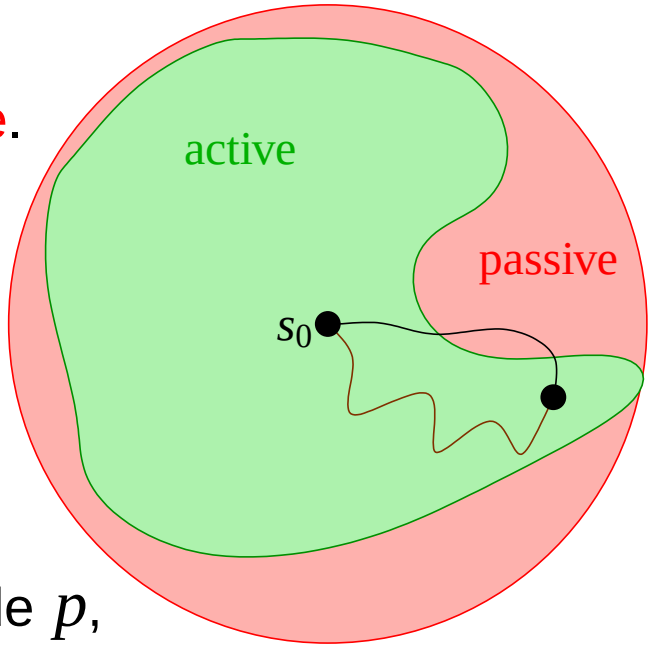
Highway Hierarchy: Level 1 and Level 2



Fast Construction

Phase 1: *Construction of Partial Shortest Path Trees*

For each node s_0, perform an SSSP search from s_0.

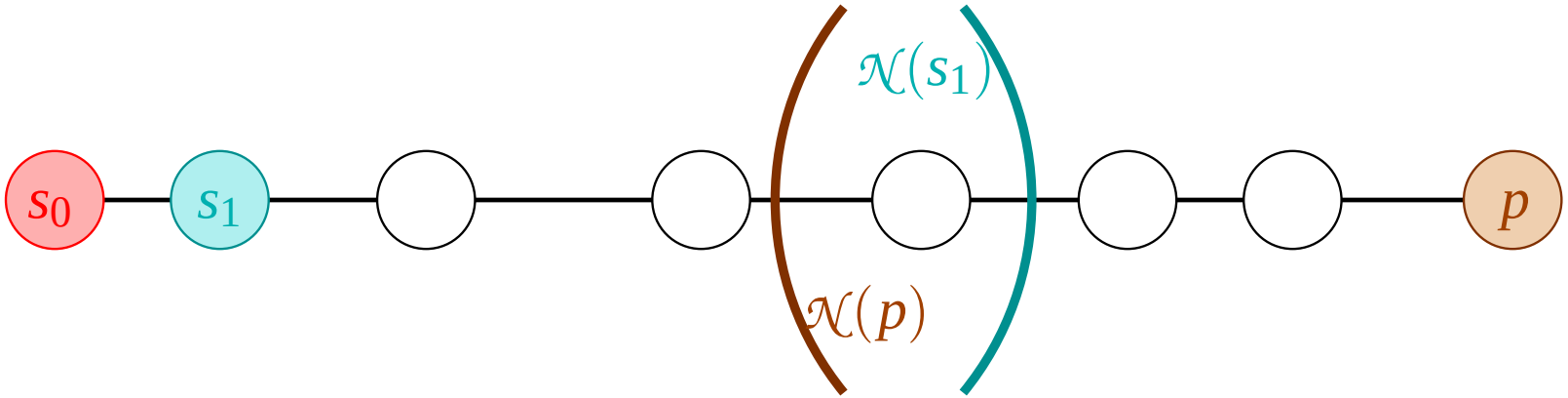


- A node's state is either active or passive.
- s_0 is active.
- A node inherits the state of its parent in the shortest path tree.
- If the abort condition is fulfilled for a node p, p's state is set to passive.

The search is aborted when all queued nodes are passive.

Fast Construction

Abort Condition:



p is set to passive *iff*

$$|\mathcal{N}(s_1) \cap \mathcal{N}(p)| \leq 1$$

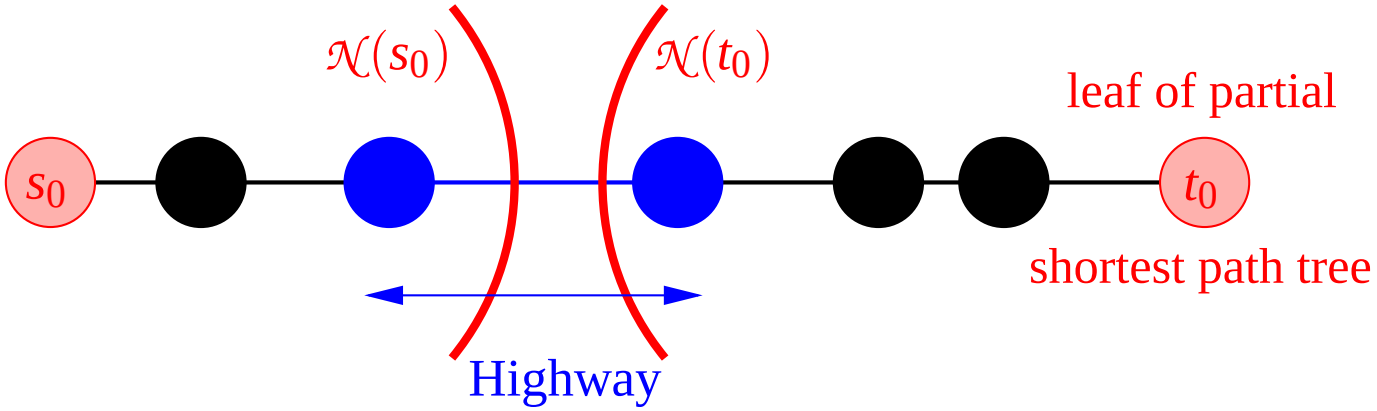
Fast Construction, Phase 2

Theorem:

The tree roots and leaves encountered in Phase 1

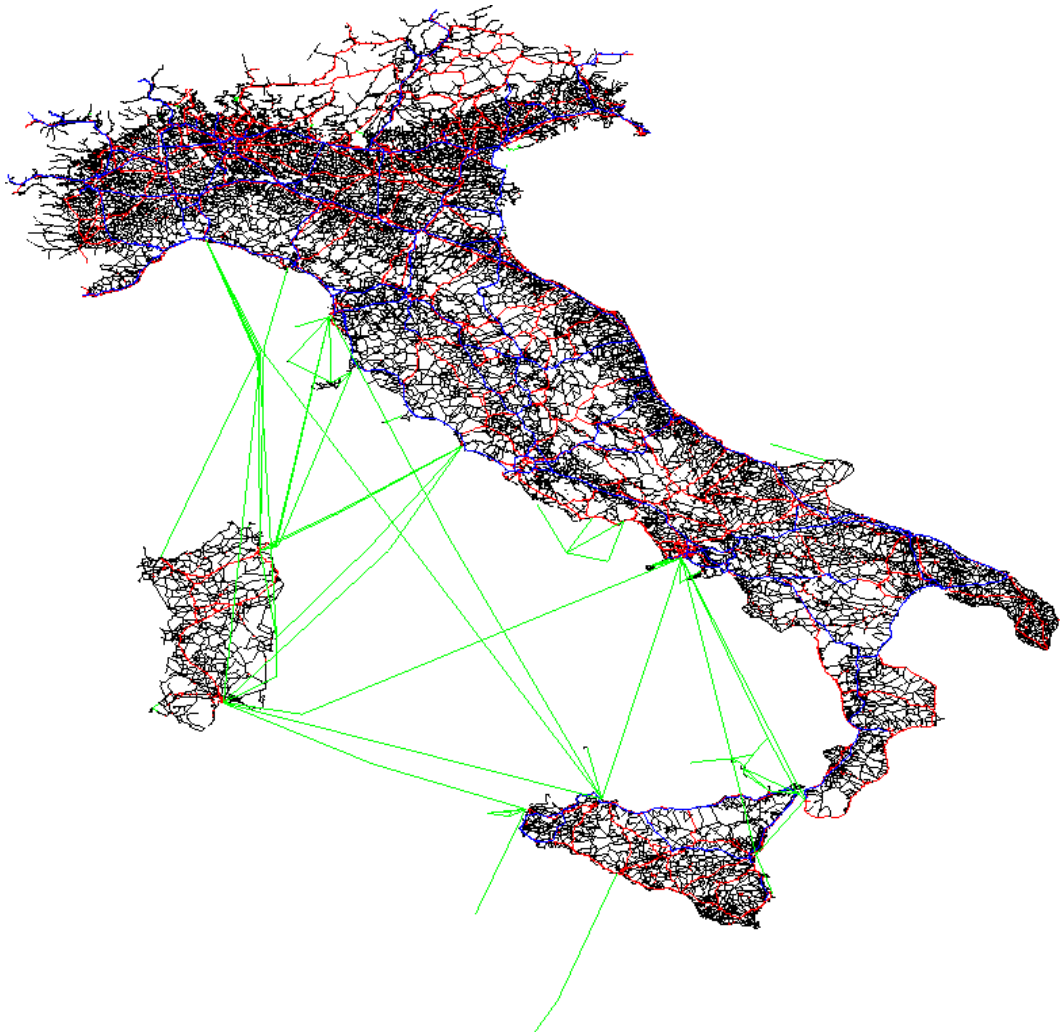
witness all highway edges.

The highway edges can be found in time linear in the tree sizes.



Fast Construction

Problem: very long edges, e.g. ferries



Faster Construction

Solution: An active node v is declared to be a maverick if

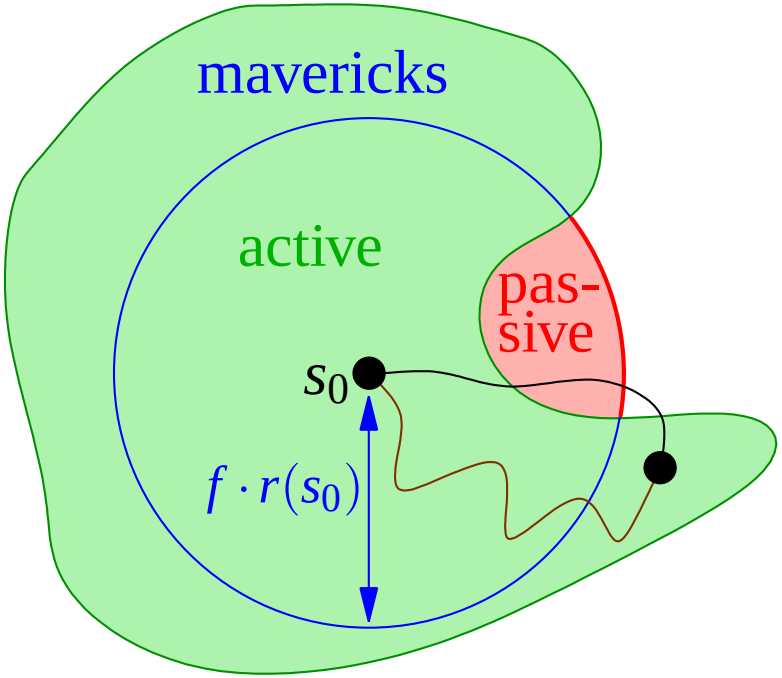
$$d(s_0, v) > f \cdot r(s_0).$$

When all active nodes are mavericks,

the search from passive nodes is

no longer continued.

$\rightsquigarrow$ superset of the highway network



Space Consumption

Choose neighborhood sizes such that levels shrink geometrically

$\rightsquigarrow$ linear space consumption

Arbitrarily Small Constant Factor (not implemented):

- Large $H_0 \rightsquigarrow$ large level-0 radius $\rightsquigarrow$ small higher levels
- No $r(\cdot)$ needed for level-0 search (under certain assumptions)
- Mapping to next level by hash table

Query

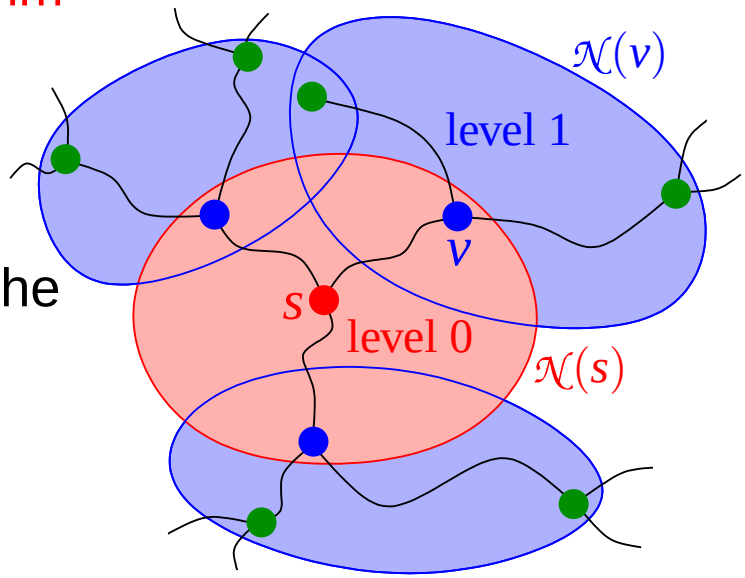
Bidirectional version of Dijkstra's Algorithm

Restrictions:

- Do not leave the neighbourhood of the entrance point to the current level.

  Instead: switch to the next level.

- Do not enter a component of bypassed nodes.



- entrance point to level 0
- entrance point to level 1
- entrance point to level 2

Query (bidir. Dijkstra I)

Operations on two priority queues $\overrightarrow{Q}$ and $\overleftarrow{Q}$:

- void **insert**(nodeID, key)
- void **decreaseKey**(nodeID, key)
- nodeID **deleteMin**()

node u has key $\delta(u)$

- (tentative) distance from the respective source node

Query (bidir. Dijkstra II)

```
query(s,t) {
    →Q.insert(s,0); ←Q.insert(t,0);
    while (→Q ∪ ←Q ≠ ∅) do {
        ⇋ ∈ {→,←};                    //select direction
        u := ⇋Q.deleteMin();
        relaxEdges(⇋,u);
    }
}
```

Query (bidir. Dijkstra III)

relaxEdges$(\overleftrightharpoons, u)$ {

  foreach $e = (u, v) \in \overrightarrow{\overleftrightharpoons E}$ **do** {

    k := $\delta(u)$ + $w(e)$;

    if $v \in \overleftrightharpoons Q$ **then** $\overleftrightharpoons Q$.decreaseKey(v, k); **else** $\overleftrightharpoons Q$.insert(v, k);

  }

}

Query (Hwy I)

Operations on two priority queues $\overrightarrow{Q}$ and $\overleftarrow{Q}$:

- void **insert**(nodeID, key)
- void **decreaseKey**(nodeID, key)
- nodeID **deleteMin**()

node u has key $(\delta(u), \ell(u), \text{gap}(u))$

- (tentative) distance from the respective source node
- search level
- gap to the next neighbourhood border

lexicographical order: <, <, >

Query (Hwy II)

```
query(s,t) {
    Q→.insert(s, (0,0,r0→(s))); Q←.insert(t, (0,0,r0←(t)));
    while (Q→ ∪ Q← ≠ ∅) do {
        ⇋ ∈ {→,←};                    //select direction
        u := Q⇋.deleteMin();
        relaxEdges(⇋, u);
    }
}
```

Query (Hwy III)

relaxEdges($\overleftrightharpoons, u$) {

foreach $e = (u,v) \in \overleftrightharpoons{E}$ **do** {

gap := gap(u);

if gap = ∞ **then** gap := $r^{\overleftrightharpoons}_{\ell(u)}(u)$; // leave component

for ($\ell := \ell(u)$; $w(e) >$ gap; ℓ++, gap := $r^{\overleftrightharpoons}_{\ell}(u)$); // go "upwards"

if $\ell(e) < \ell$ **then continue**; // Restriction 1

if e "enters a component" **then continue**; // Restriction 2

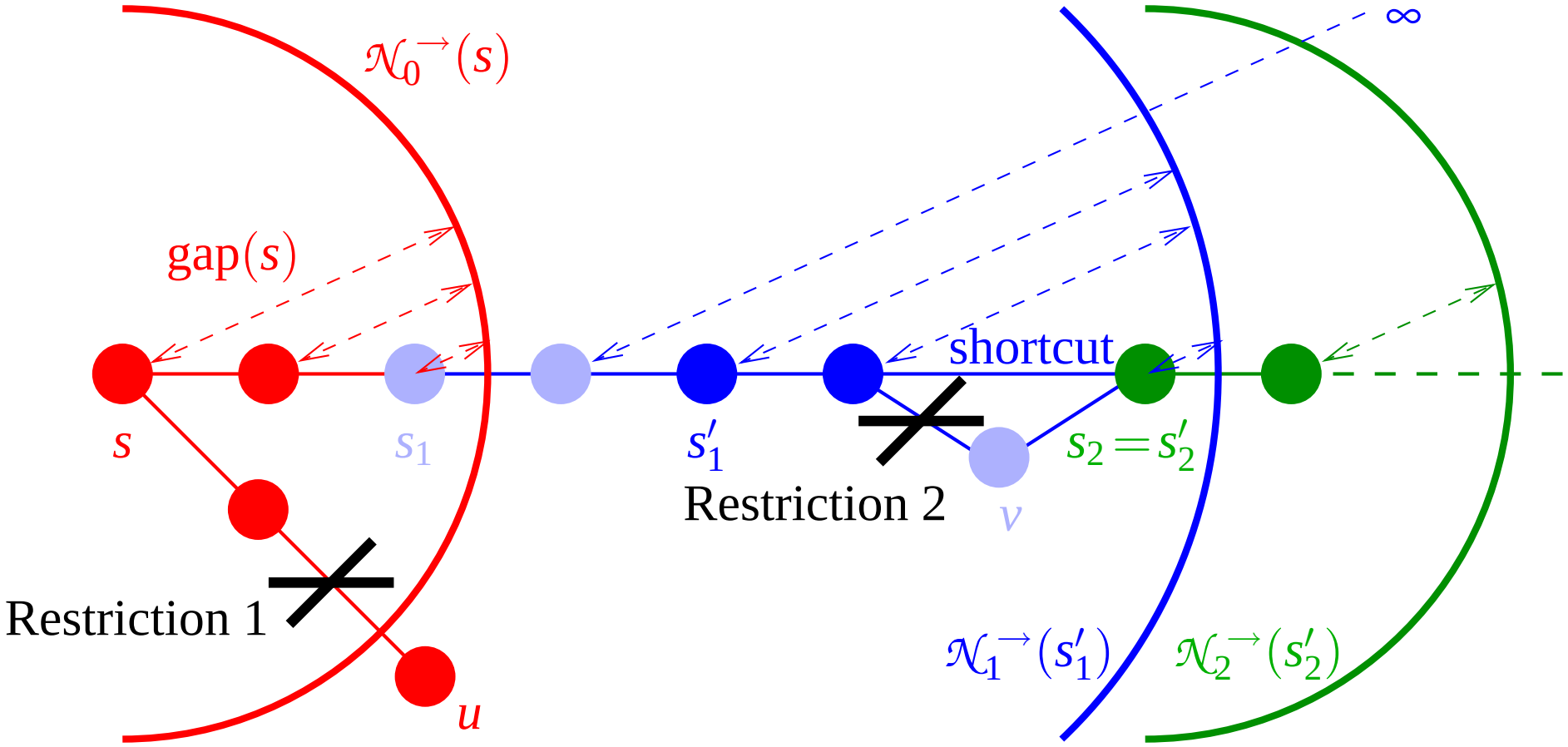
k := ($\delta(u) + w(e)$, ℓ, gap$-w(e)$);

if $v \in \overleftrightharpoons{Q}$ **then** $\overleftrightharpoons{Q}$.decreaseKey($v,k$); **else** $\overleftrightharpoons{Q}$.insert($v,k$);

}

}

Query


$\mathcal{N}_0^{\rightarrow}(s)$
gap(s)
∞
shortcut
s
s_1
s_1'
$s_2 = s_2'$
Restriction 2
v
Restriction 1
u
$\mathcal{N}_1^{\rightarrow}(s_1')$
$\mathcal{N}_2^{\rightarrow}(s_2')$

Query

Theorem:

We still find the shortest path.

Query

Example: from Karlsruhe, Am Fasanengarten 5

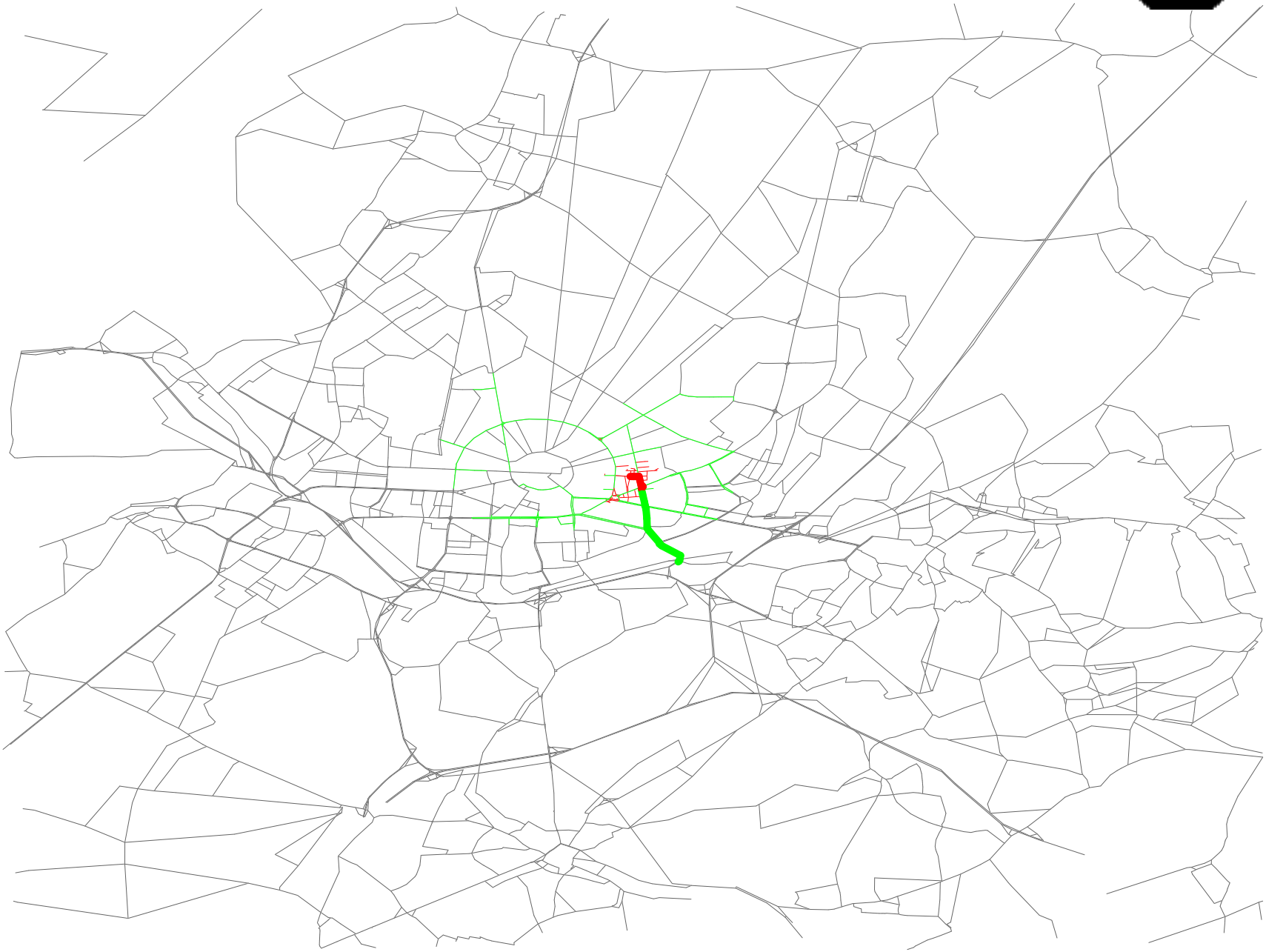
to Palma de Mallorca

Bounding Box: 20 km Level 0

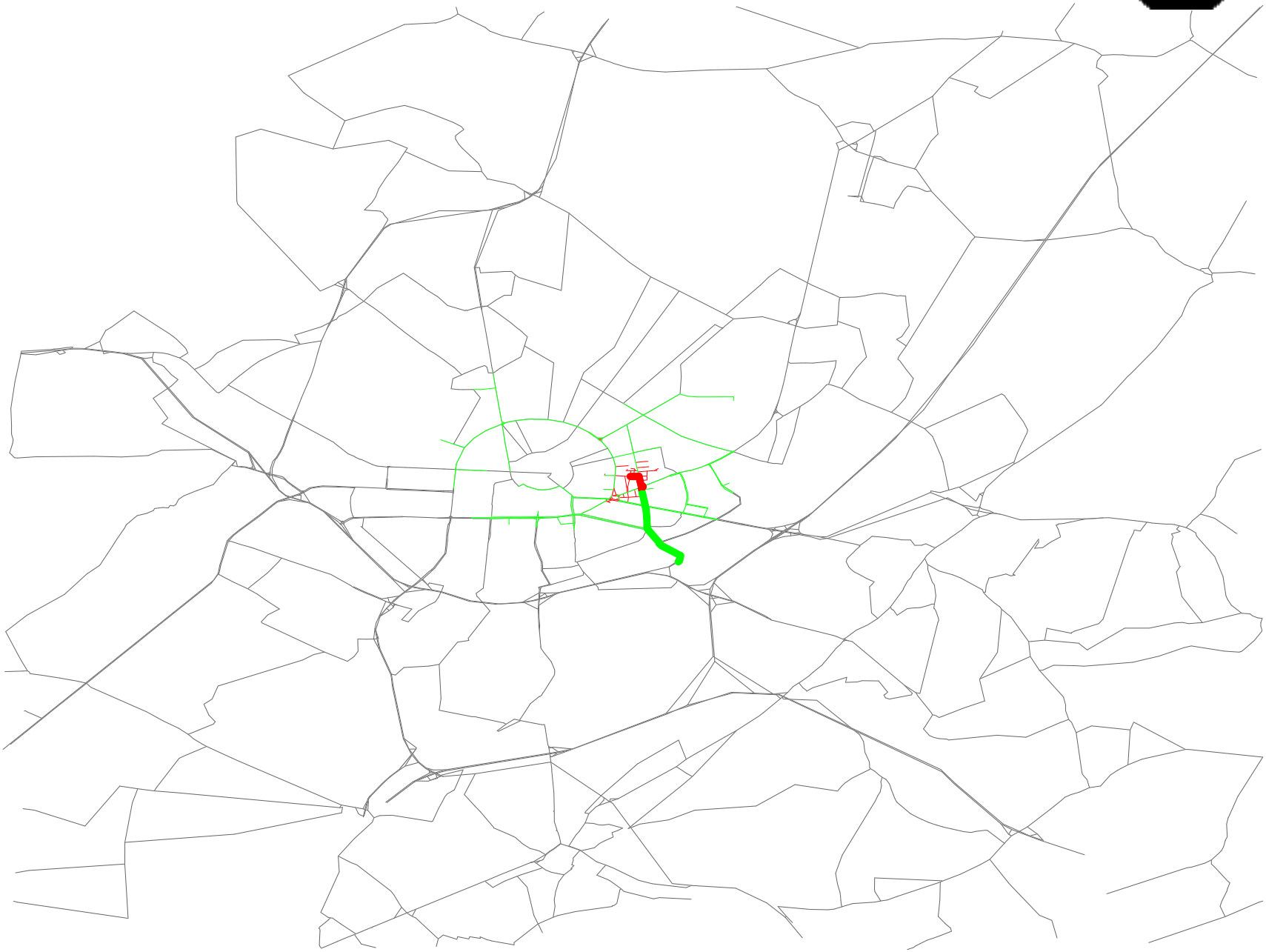
Bounding Box: 20 km Level 0 Search Space

Bounding Box: 20 km Level 1

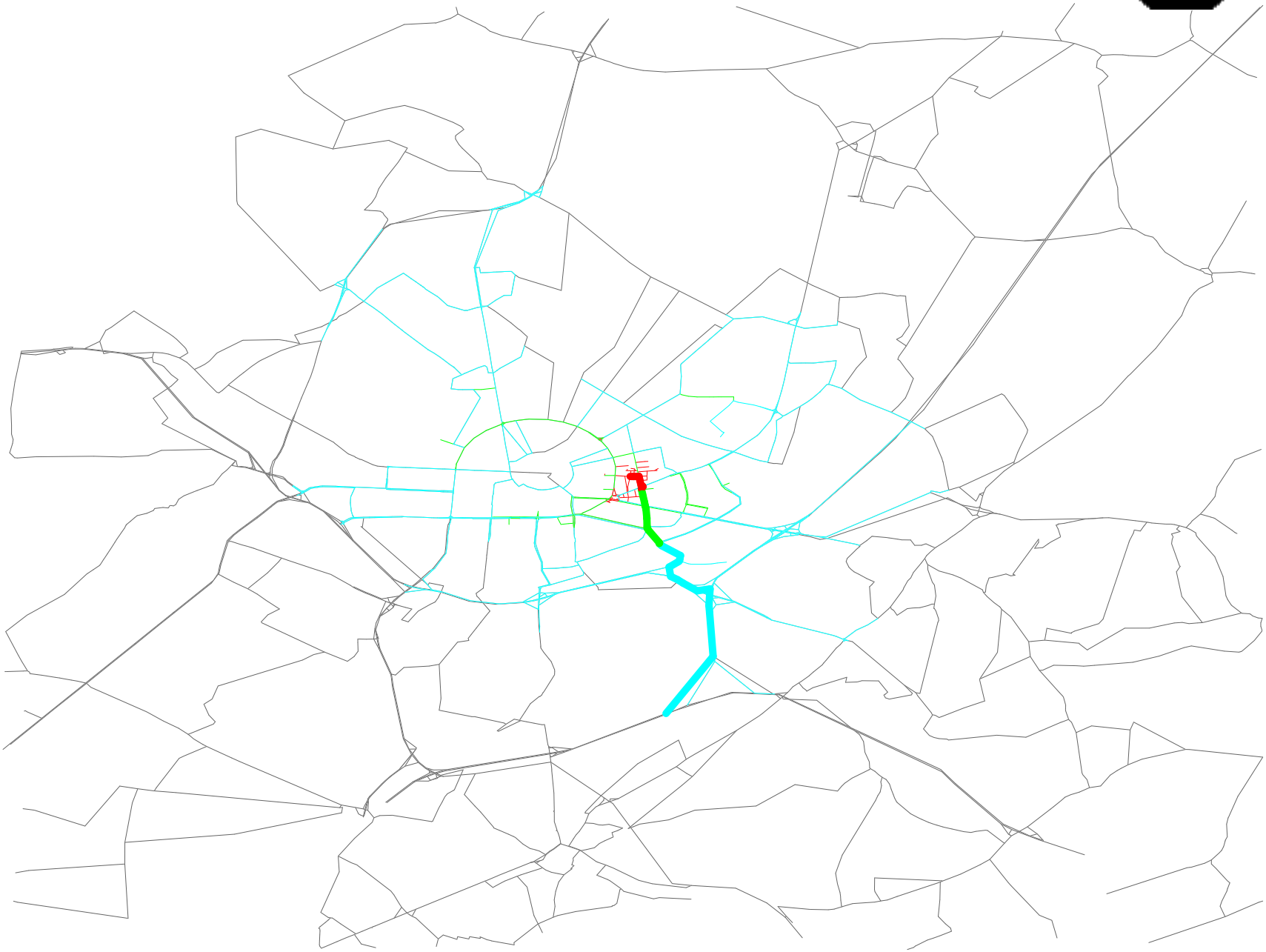
Bounding Box: 20 km Level 1 Search Space



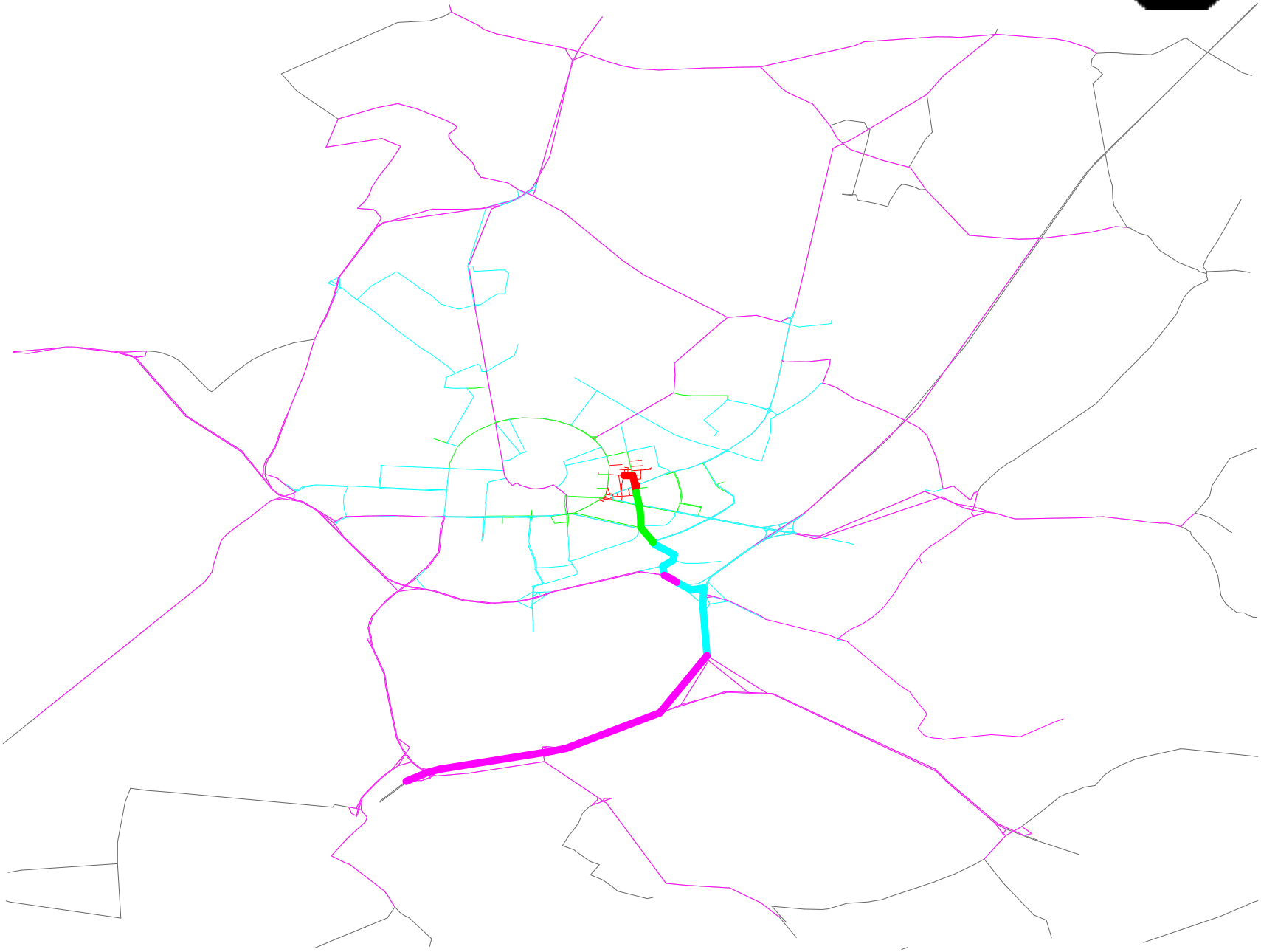
Bounding Box: 20 km Level 2



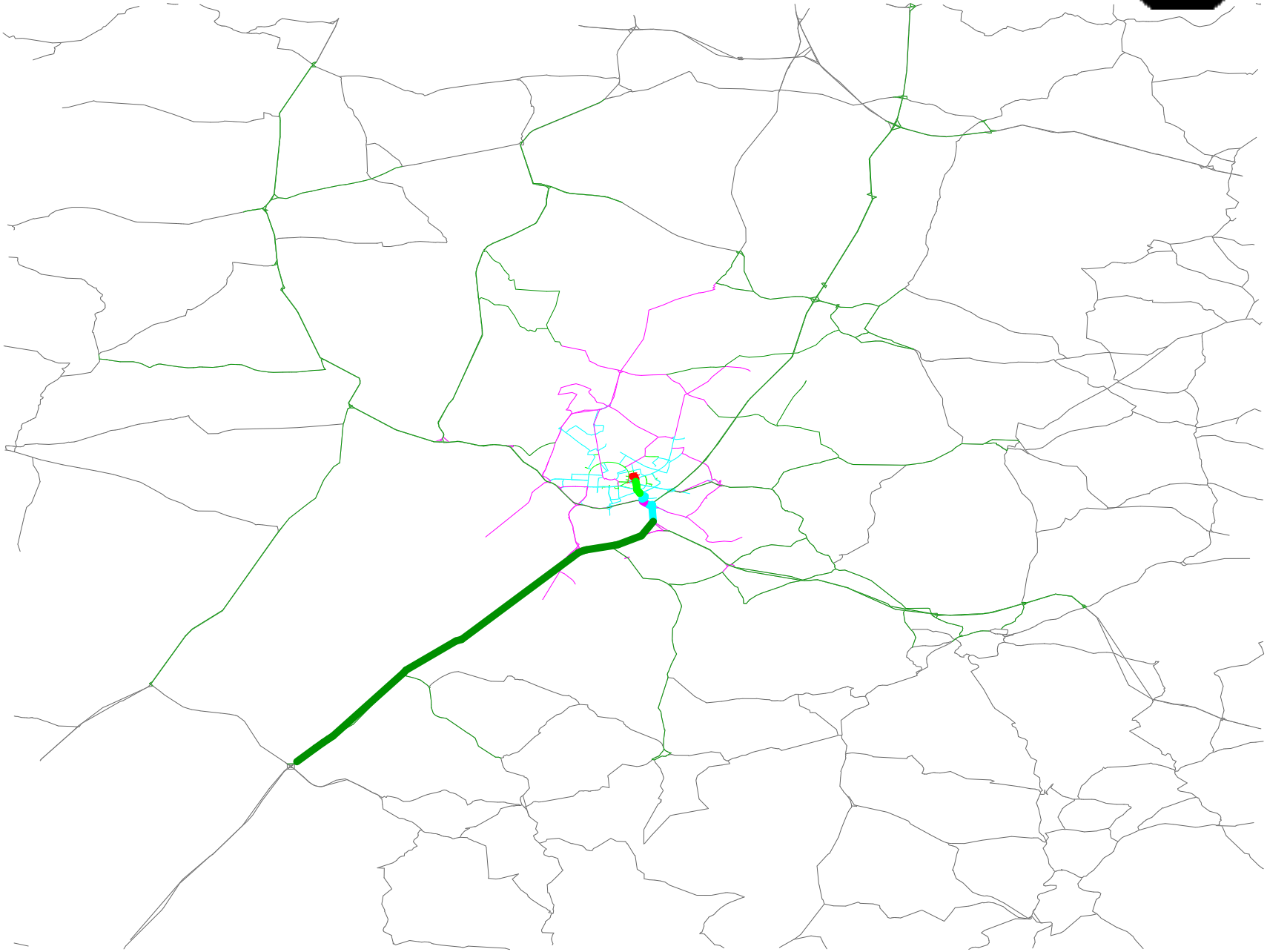
Bounding Box: 20 km Level 2 Search Space



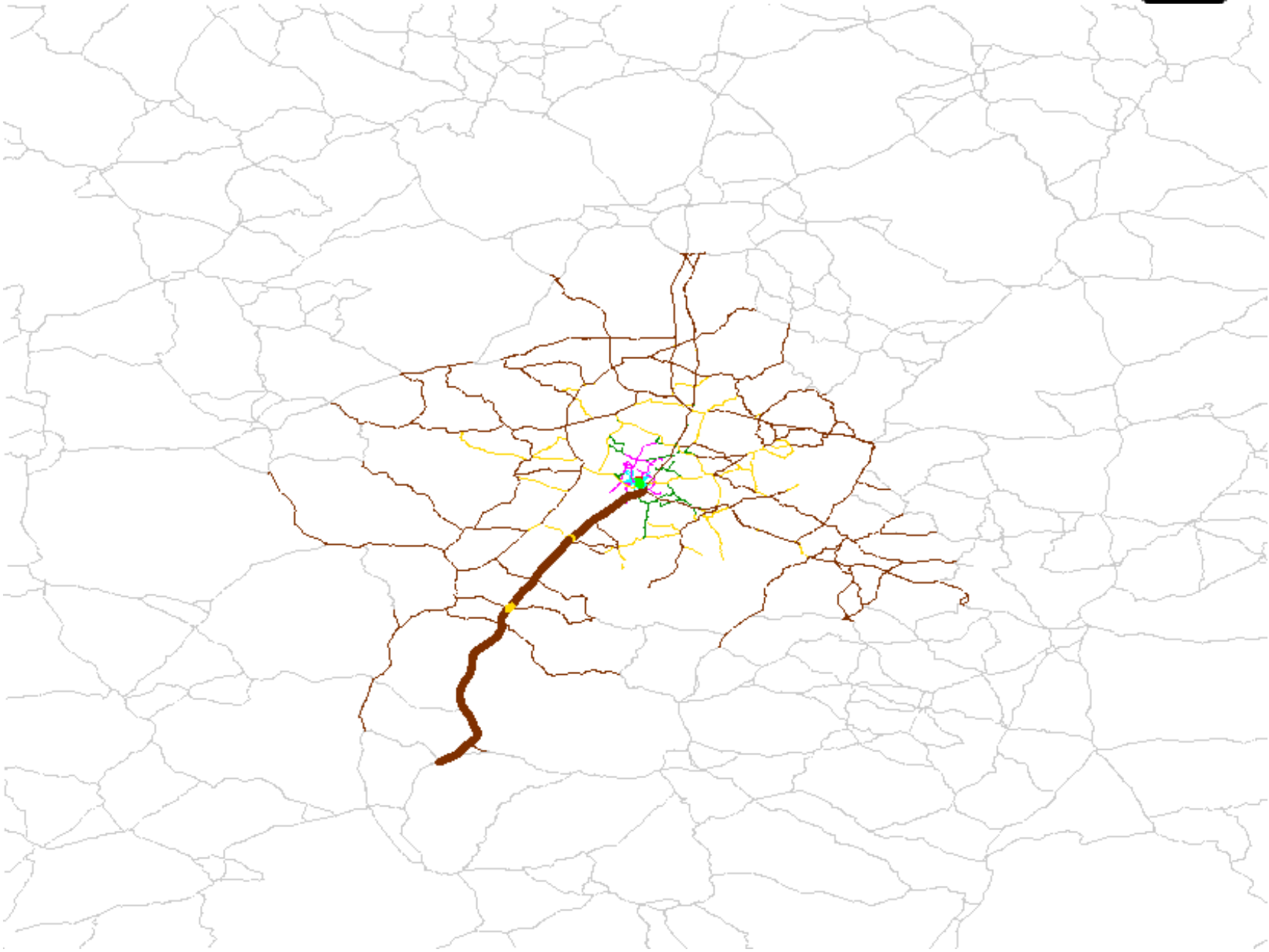
Bounding Box: 20 km Level 3 Search Space



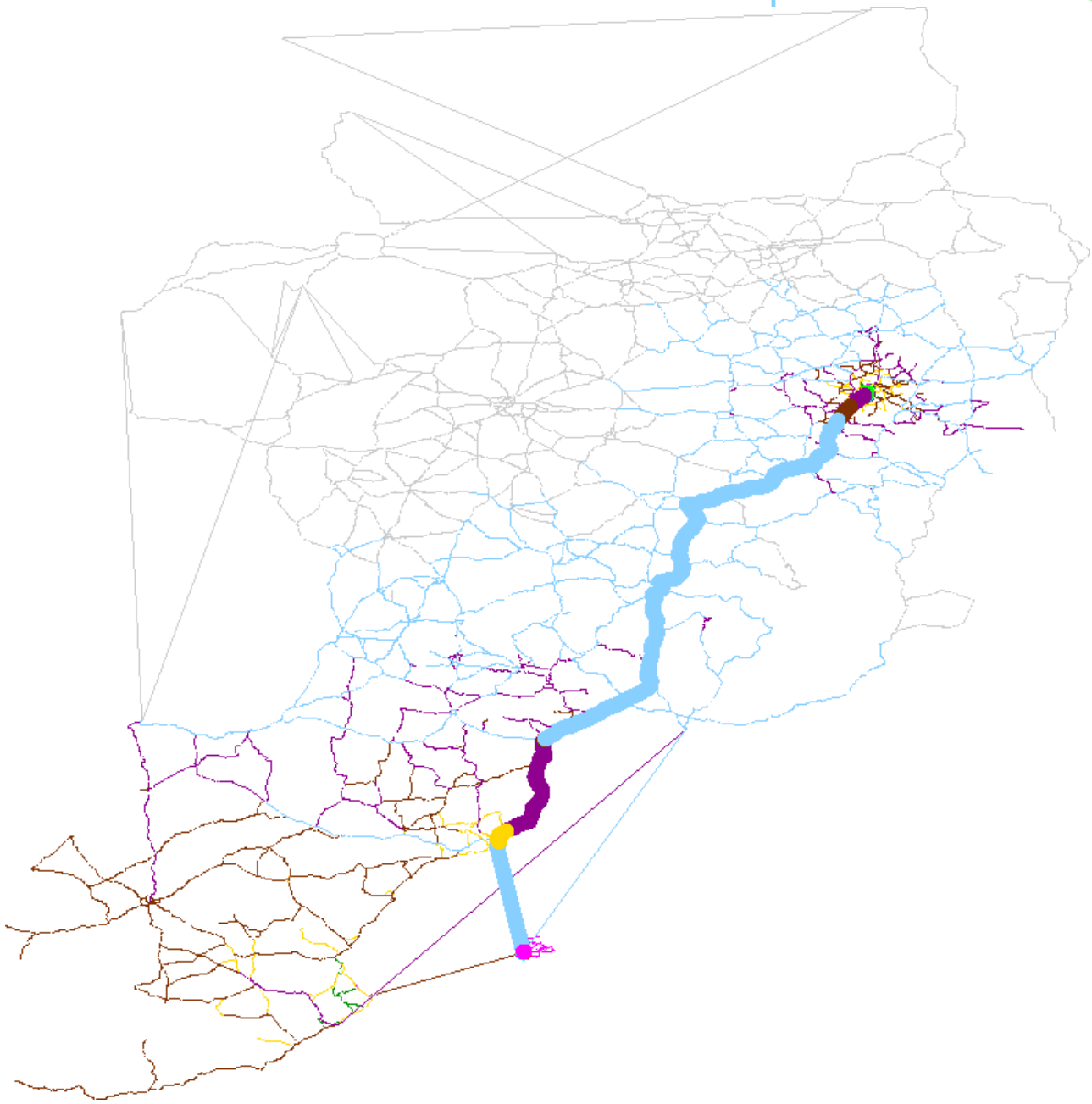
Bounding Box: 80 km Level 4 Search Space



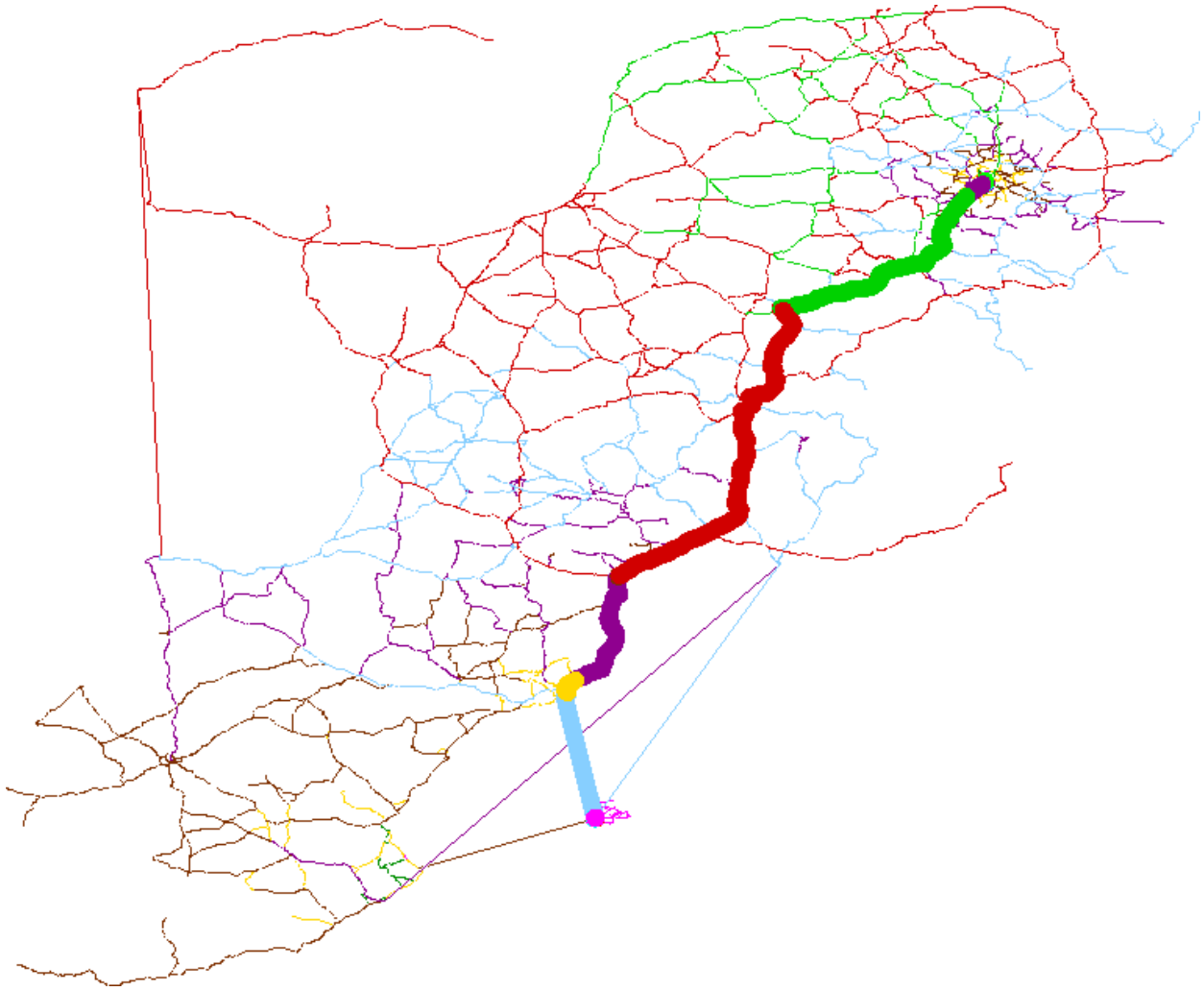
Bounding Box: 400 km Level 6 Search Space



Level 8 Search Space



Level 10 Search Space



Optimisation: Distance Table

Construction:

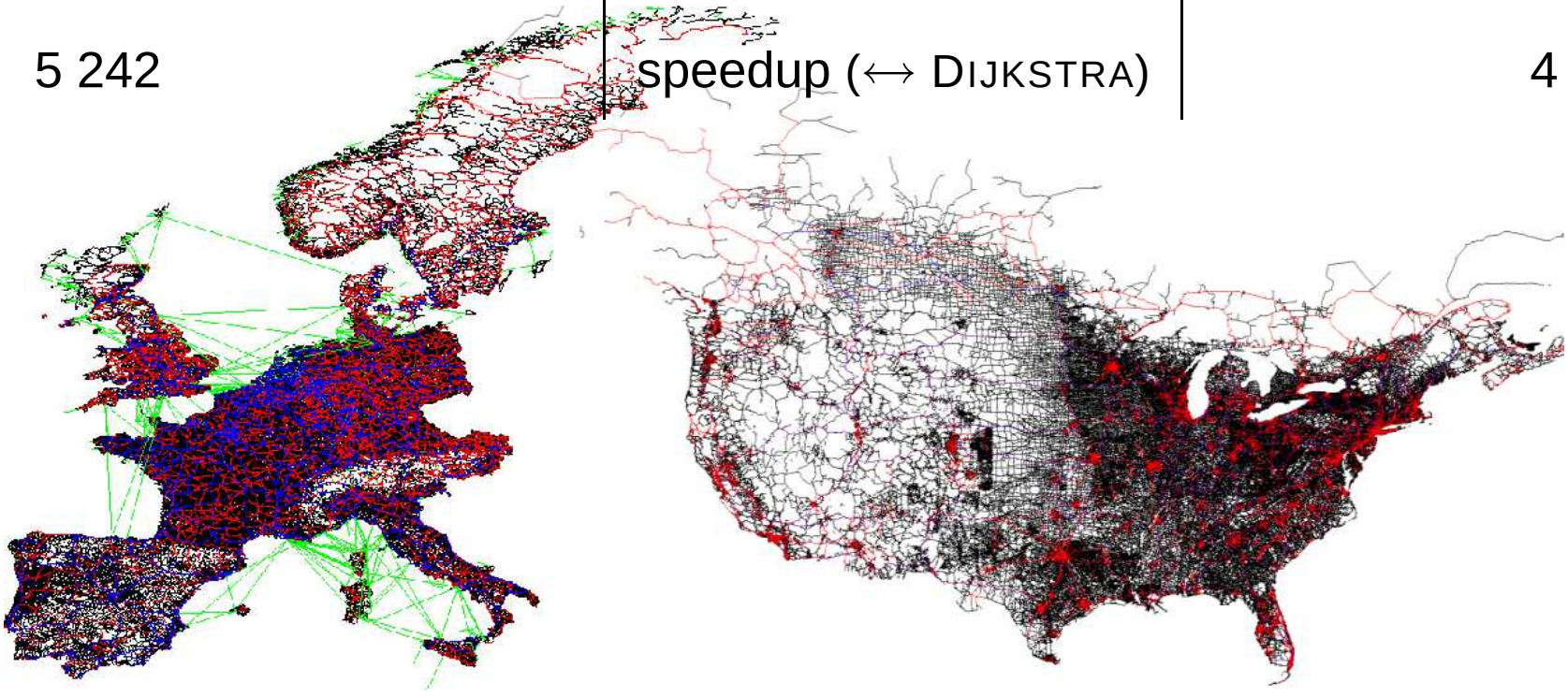
- Construct fewer levels. — e.g. 4 instead of 9
- Compute an all-pairs distance table for the topmost level L. — 8 776 $\times$ 8 776 entries

Query:

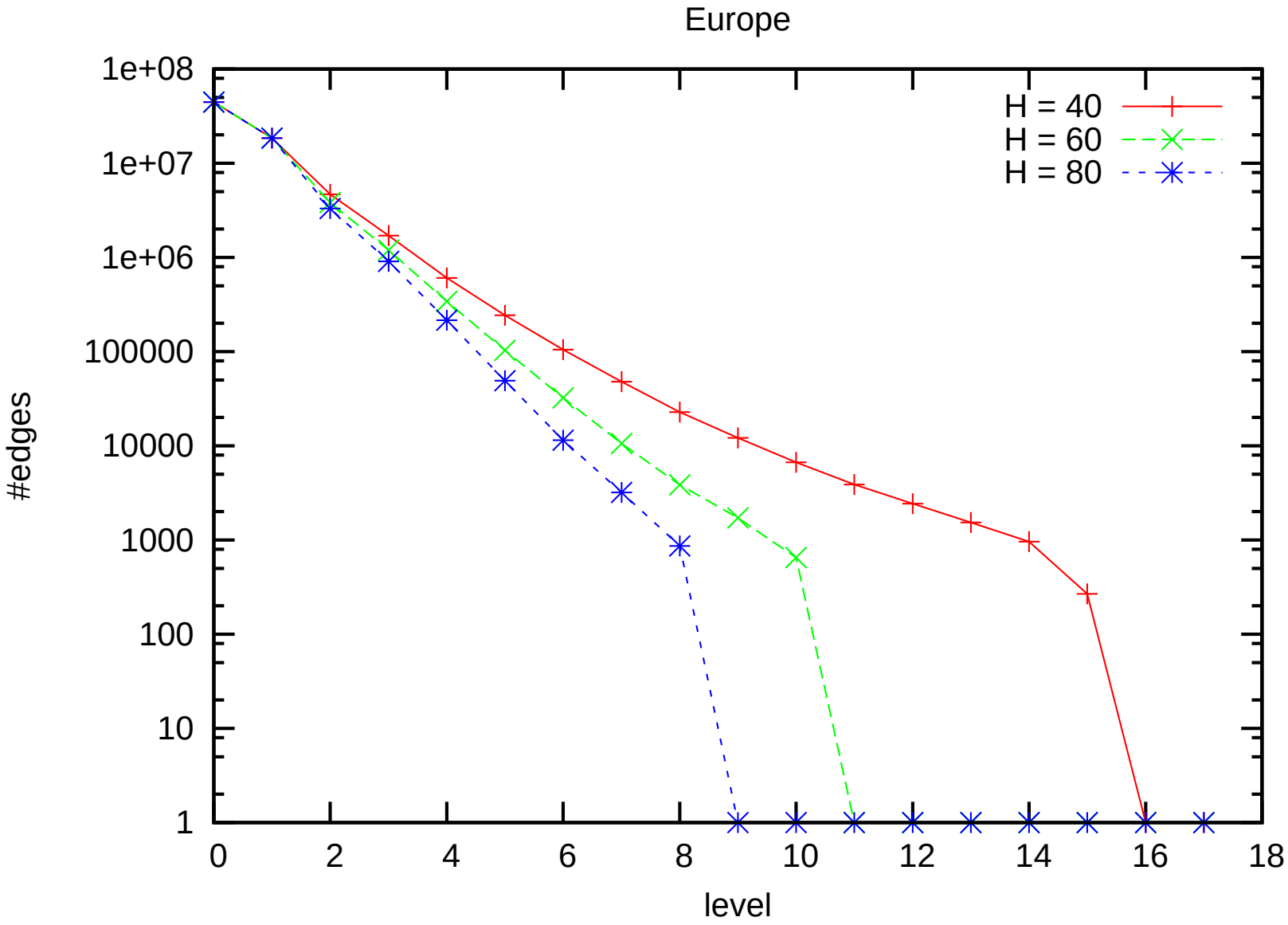
- Abort the search when all entrance points in the core of level L have been encountered. — $\approx$ 70 for each direction
- Use the distance table to bridge the gap. — $\approx$ 70 $\times$ 70 entries

Experiments

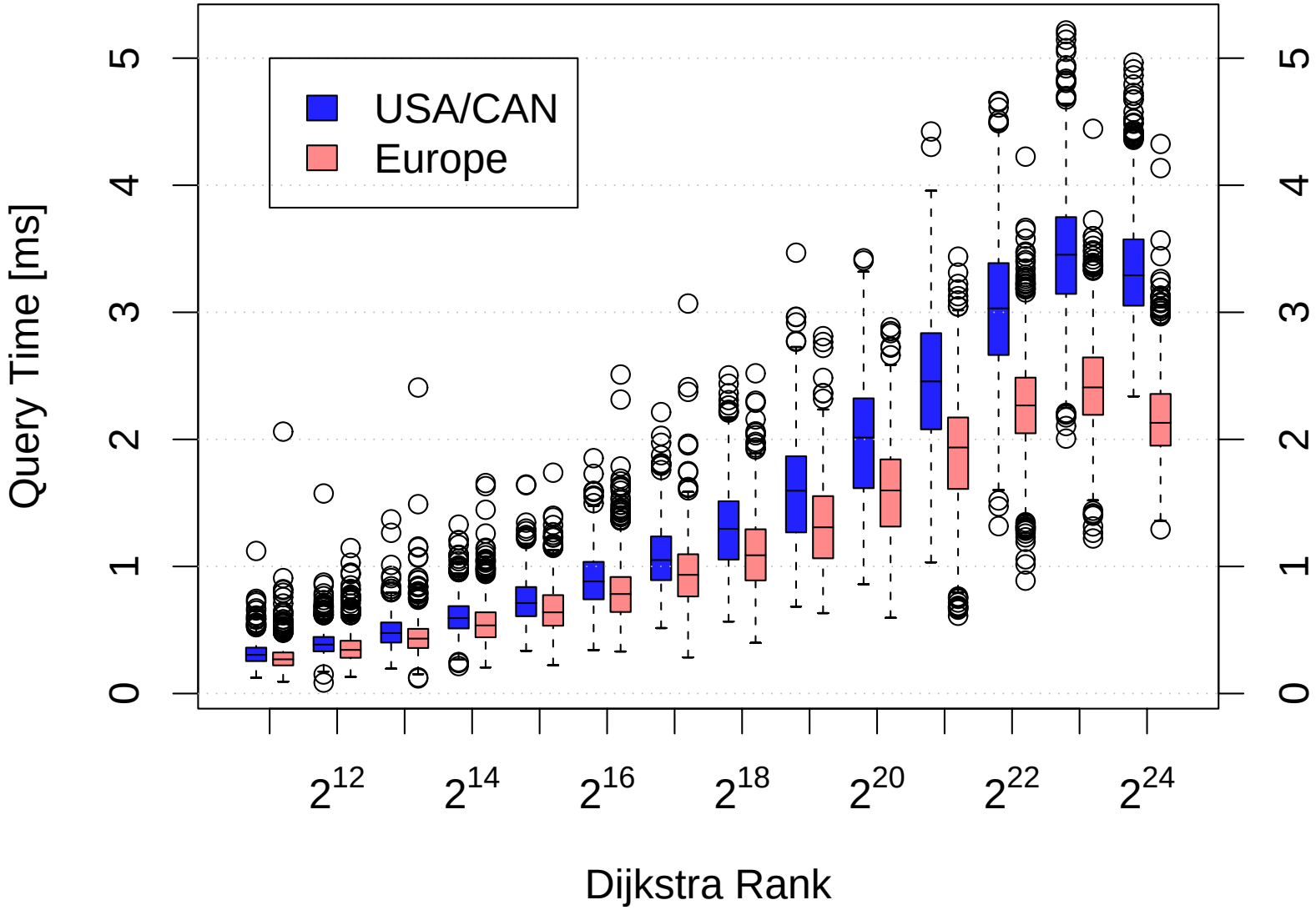
W. Europe (PTV)		USA/CAN (PTV)
18 029 721	#nodes	18 741 705
42 199 587	#directed edges	47 244 849
21	construction [min]	30
1.94	search time [ms]	2.49
5 242	speedup ($\leftrightarrow$ DIJKSTRA)	4 021



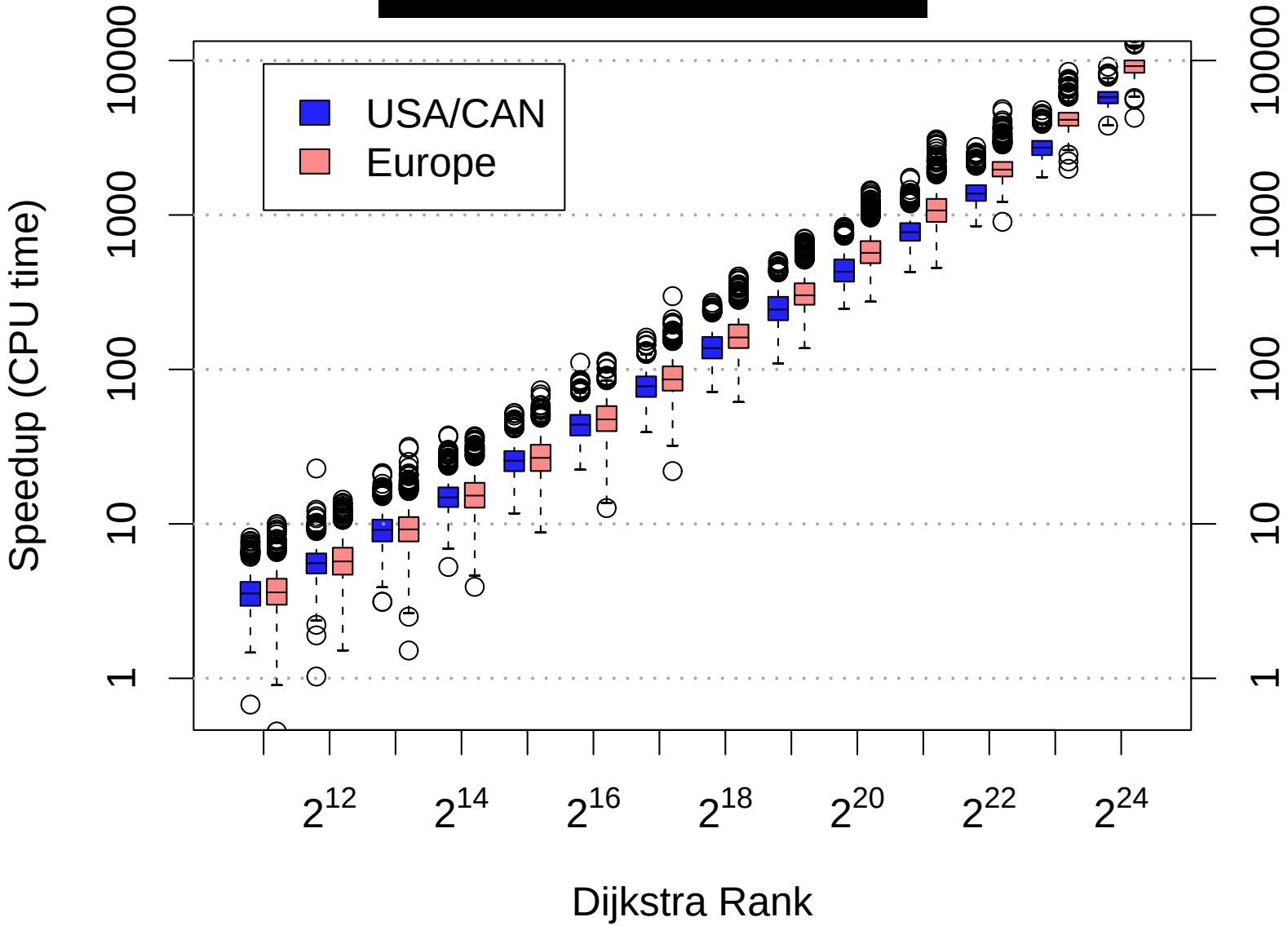
Shrinking of the Highway Networks



Queries – Time



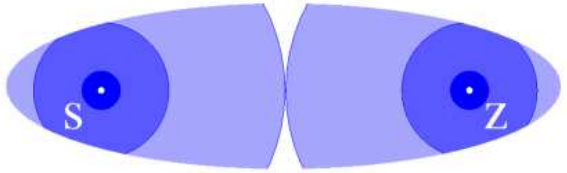
Queries – Speedup



Summary

- exact routes in large street networks — e.g. $\approx$ 18 million nodes
  - $\rightsquigarrow$ quality advantage, advertisement argument
- fast search — $\approx$ 2.5 ms
  - $\rightsquigarrow$ cheap, energy efficient processors in mobile devices
  - $\rightsquigarrow$ low server load
  - $\rightsquigarrow$ lots of room for additional functionality
- fast preprocessing — $\approx$ 30 min
- low space consumption — $\ll$ data base
- no manual postprocessing of data
  - $\rightsquigarrow$ less dependence on data sources
- organic enhancement of existing commercial solutions

Future Work



- combination with goal directed approaches
- fast, local updates on the highway network

  (e.g. for traffic jams)

- Implementation for mobile devices

  (flash access . . .)

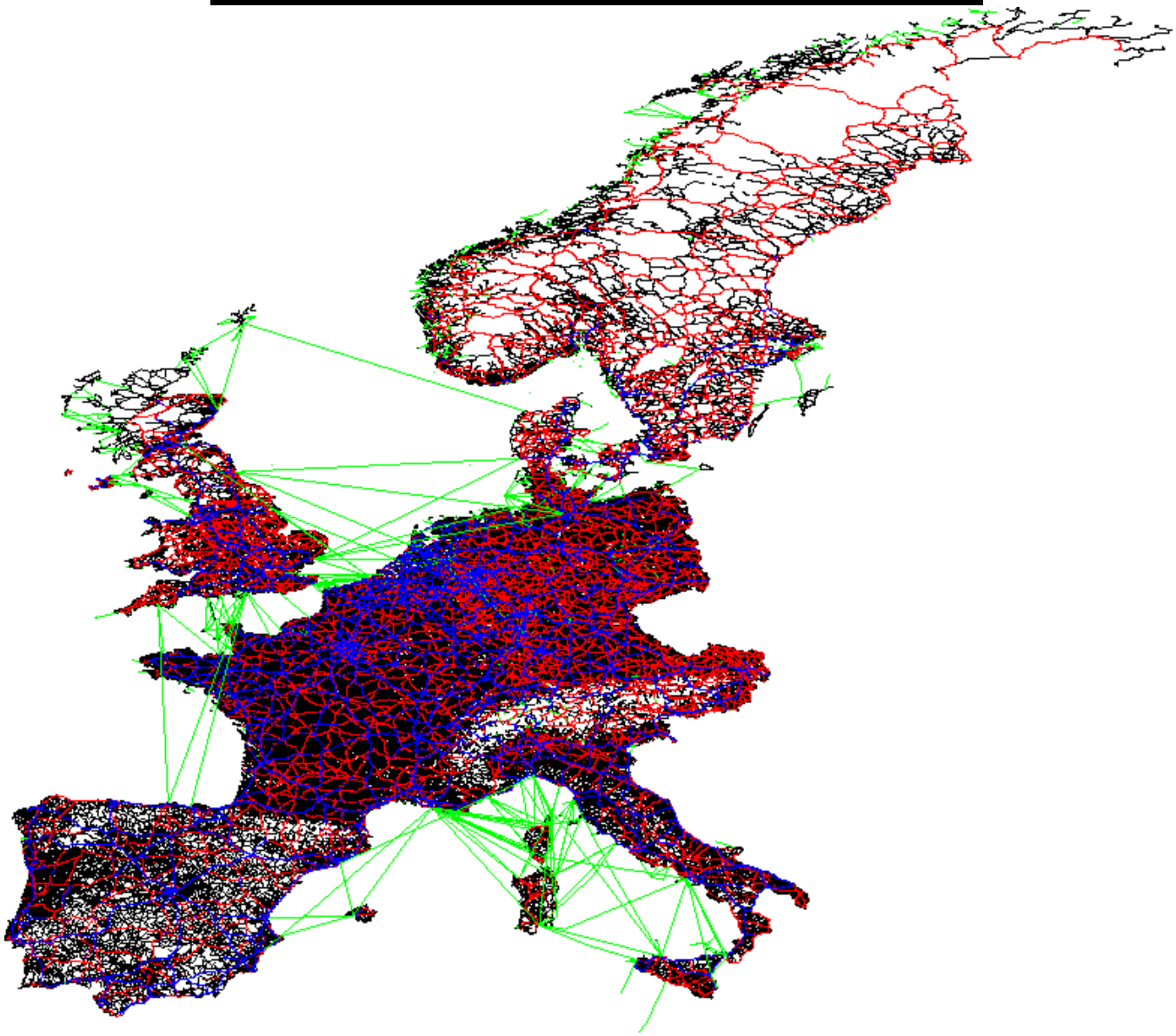
- Flexible objective functions
- . . .

Industrial Cooperations

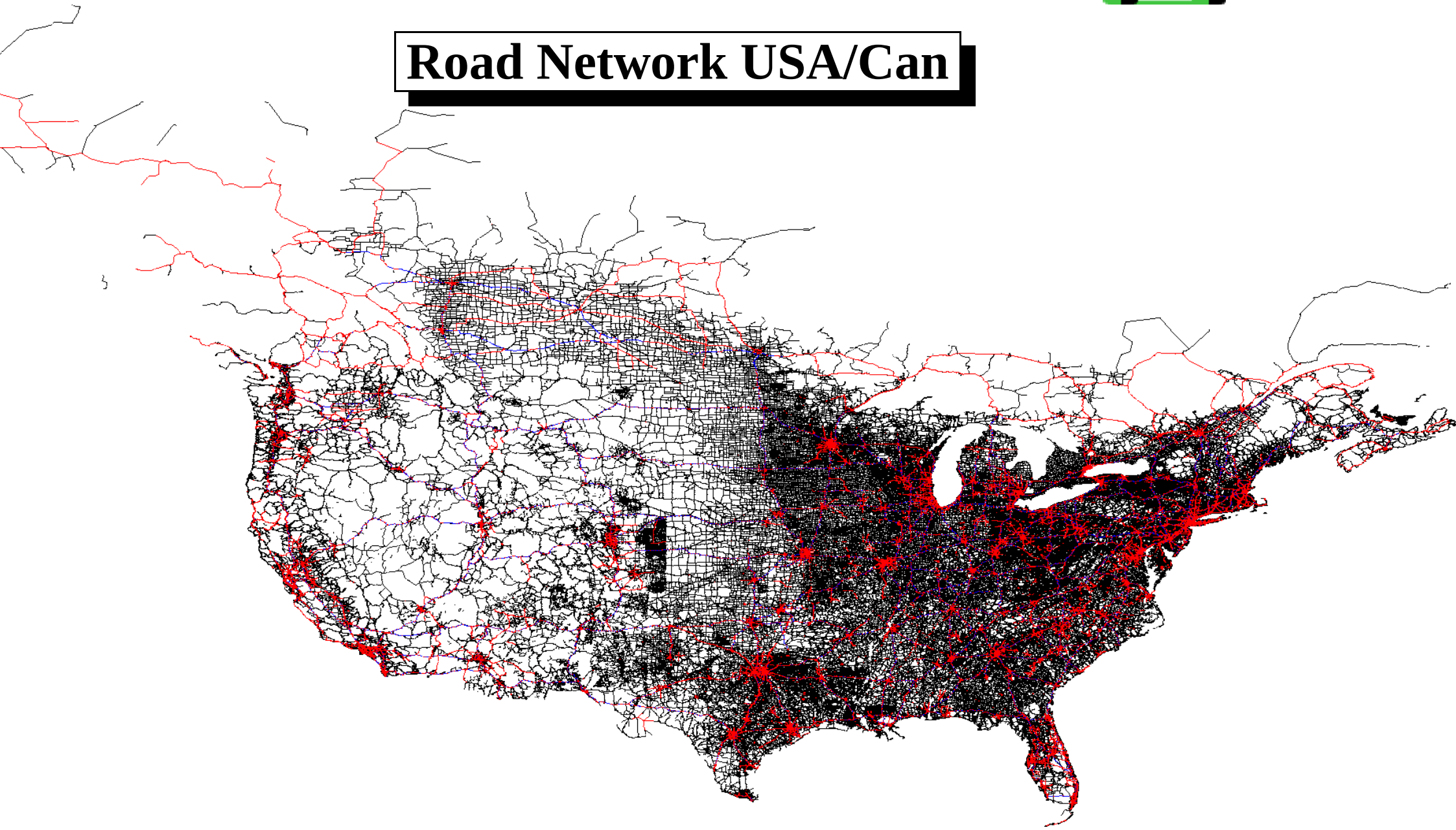
- We help transforming technology into products: consulting . . .
- Joint projects for further features

. . .

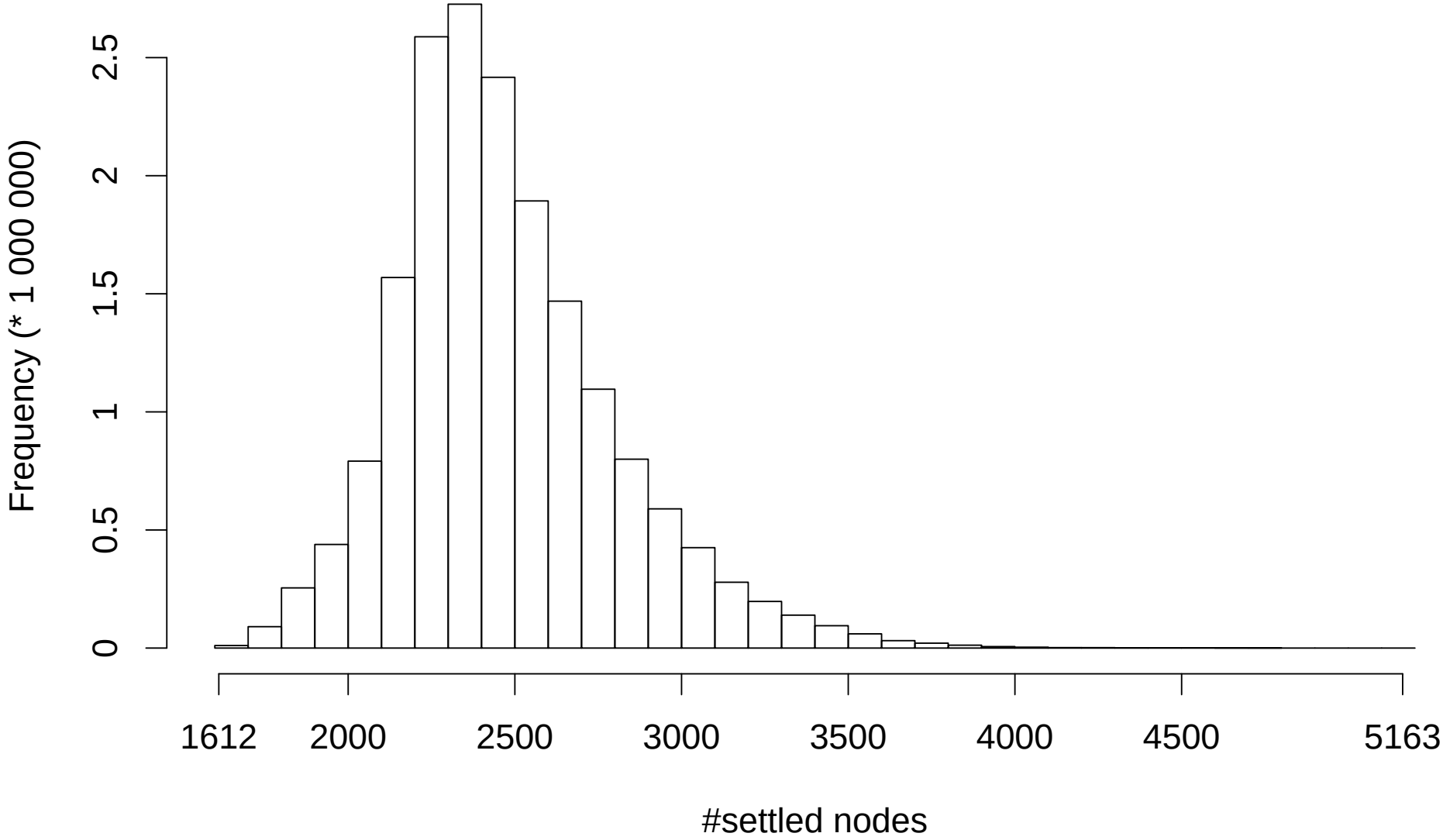
Road Network of Europe



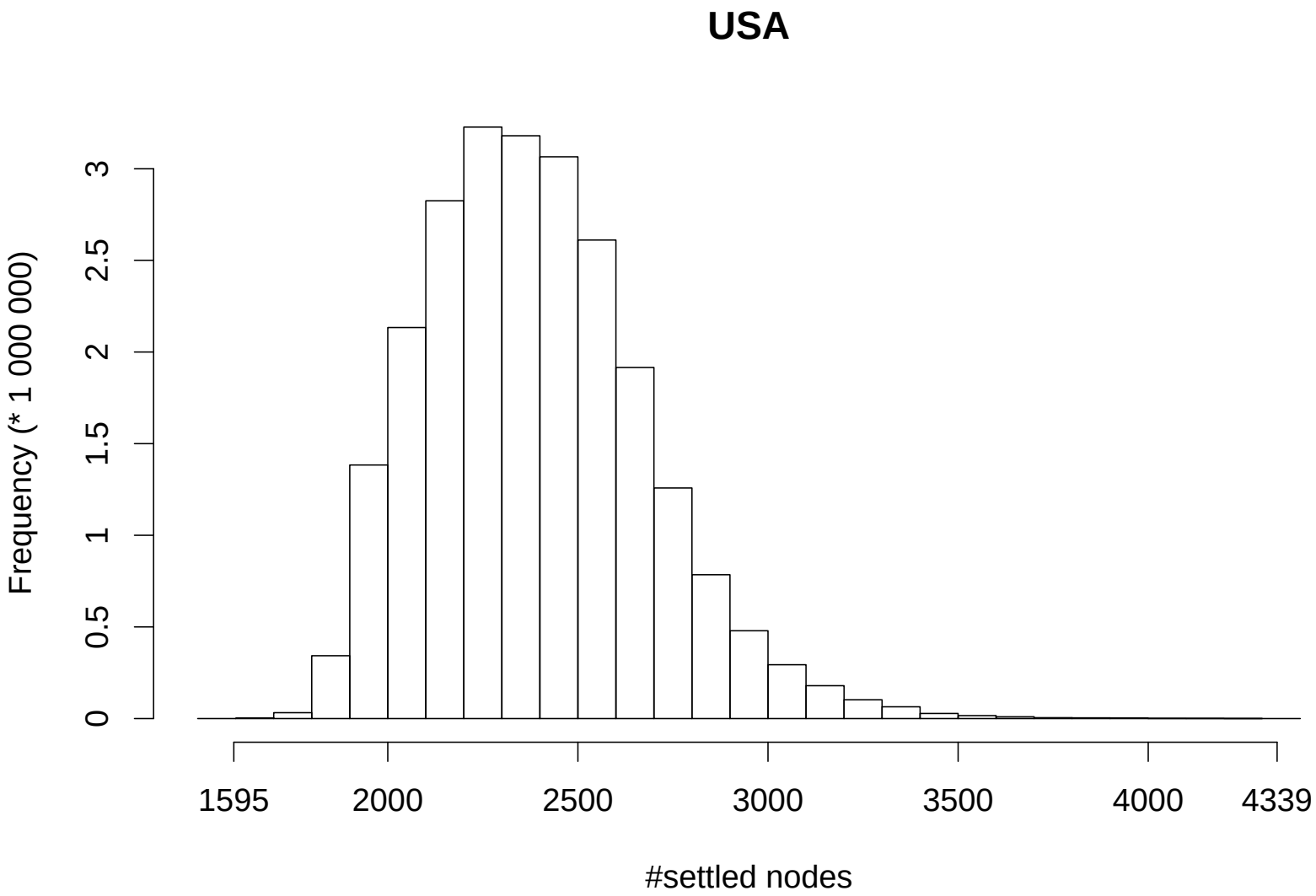
Road Network USA/Can



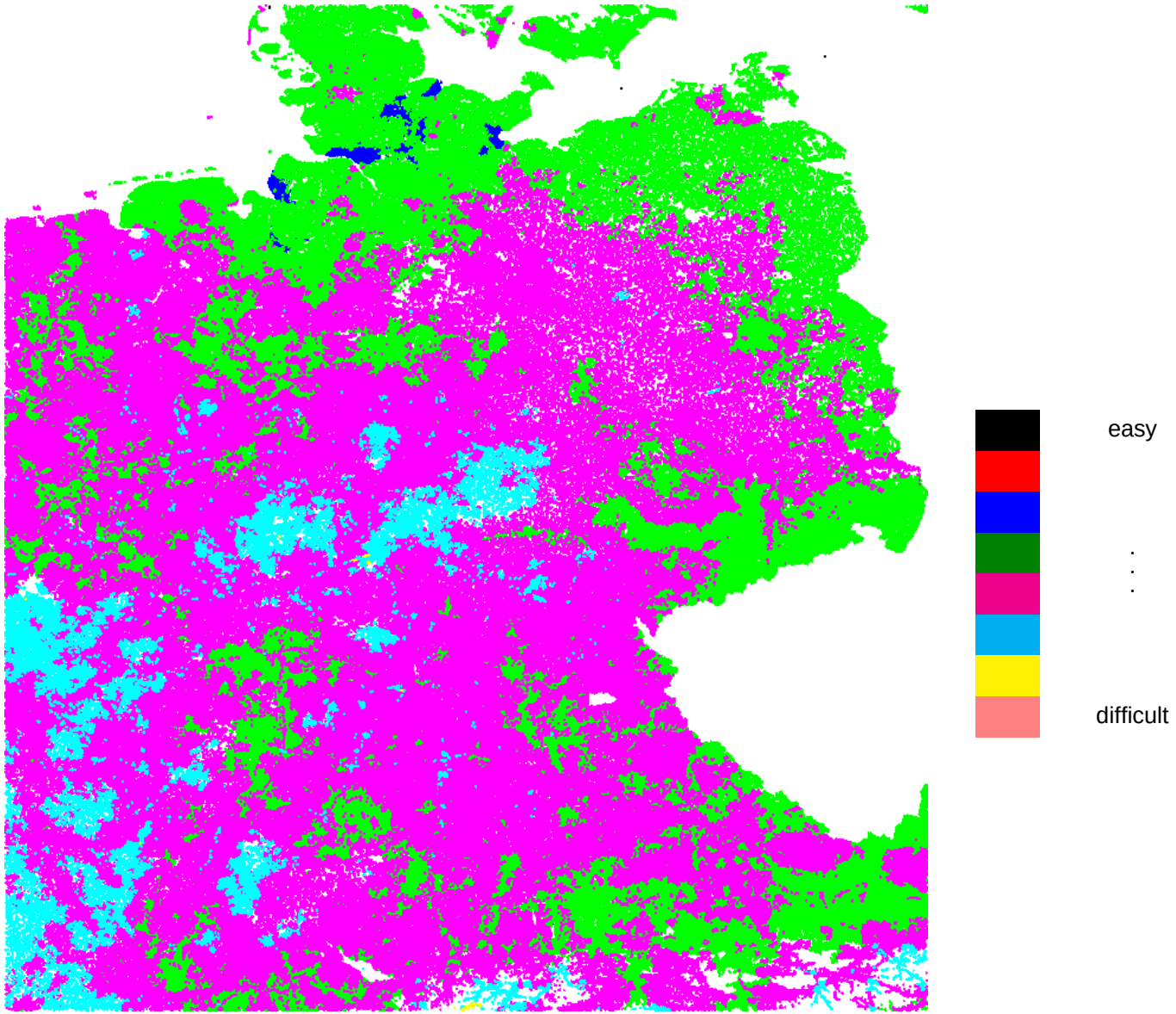
Worst Case Costs



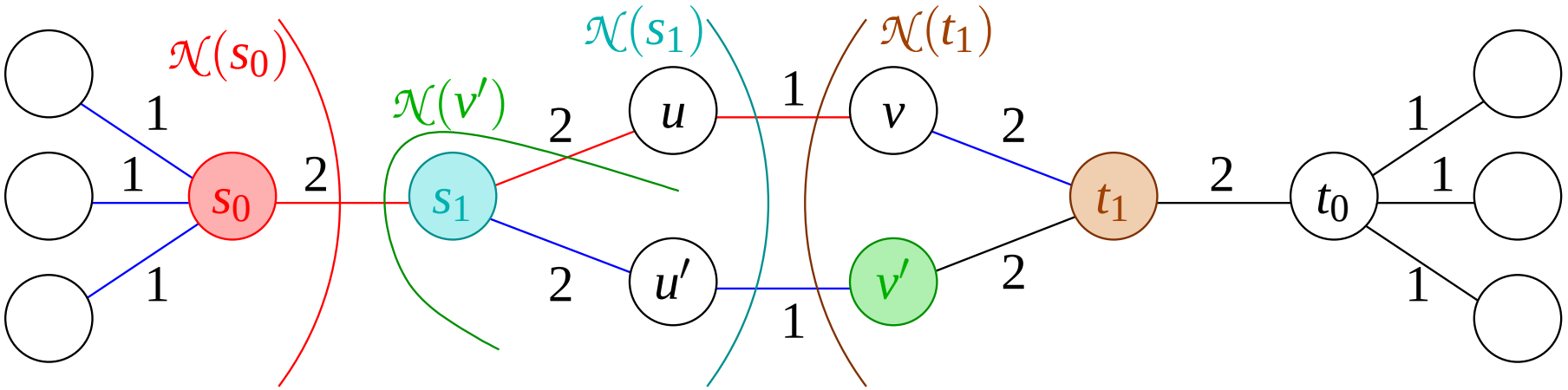
Unidirectional Search – Histogram



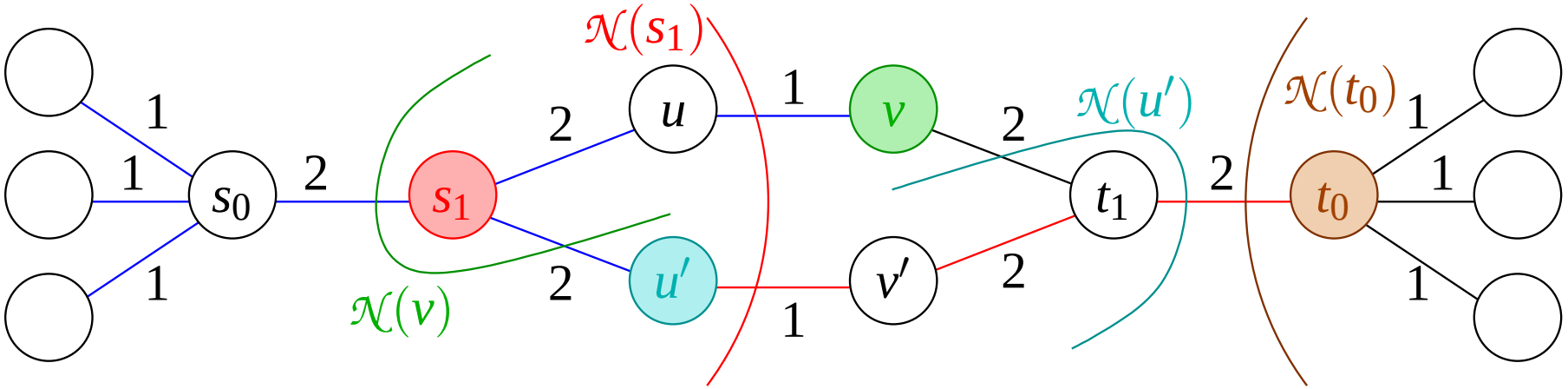
Unidirectional Search – Costs By Region



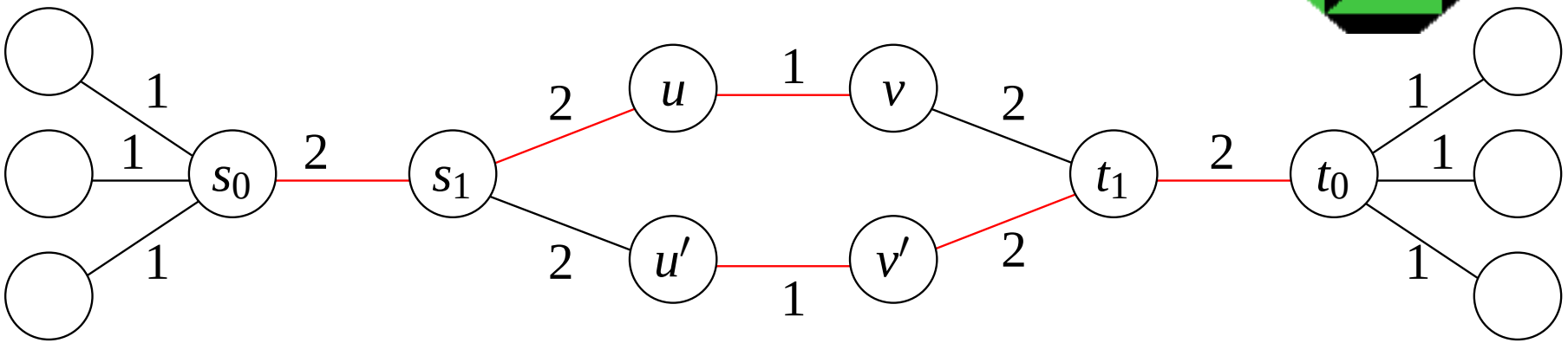
Canonical Shortest Paths



(a) Construction, started from s_0.



(b) Construction, started from s_1.



(c) Result of the construction.